UNIT-II

ELECTRO STATICS - II

Electric potential - Workdone in moving a point charge in an electro static field, Electric potential, properties of potential function, potential gradient. Conductors and Dielectrics - Conductors - Current, Current density, Equation of Continuity, Conduction Current, Ohm's law in point form, Behaviour of Conductors in an electric field, Dielectrics - polarization, Displacement and Convection current, electric field inside a dielectric material. Electric dipole - Dipole moment, potential and electric field intensity due to an electric dipole.

Boundary conditions: Conductor - Free space and Dielectric - Dielectric boundary Conditions.

Capacitance - Capacitance of parallel plate and Spherical and Co-axial Capacitors with Composite dielectrics. Laplace's and poisson's equation - Solution of Laplace's equation in one Variable. Capacitance calculation in static electric field.

ELECTRIC POTENTIAL (OR) ELECTRIC SCALAR POTENTIAL (OR) ABSOLUTE ELECTRIC POTENTIAL

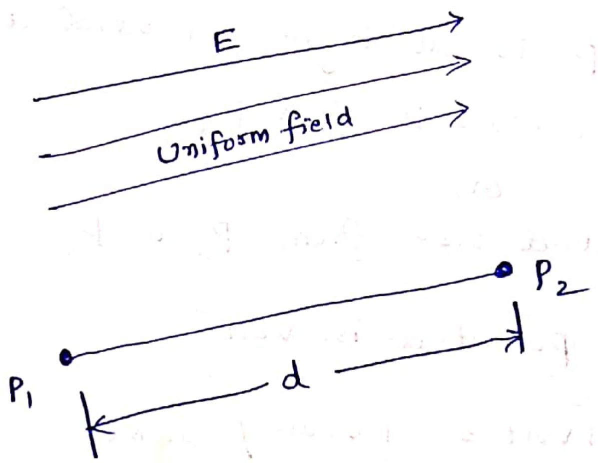

Fig: A test charge is moved from P_2 to P_1 against the uniform field E.

Consider a uniform field $\bar{E}$ produced in a restricted region of space at a sufficiently great distance from a point charge. Although point charge produces a non-uniform field.

→ Let a positive test charge be moved from P_2 to P_1 in a direction opposite to the field-orientation. What happens? The existing electric field E exerts on a force on the charge, so that some work has to be performed in order to move the charge against the force.

The magnitude of this work is

Workdone = Force x ~~Disp to~~ Distance

$= |E| q \cdot d$

where q is test charge.

Hence, the workdone per unit charge is given by $|E| d$ Joules/Coulomb

↑ Electric potential.

→ The work or energy per unit charge required to move the test charge from P_2 to P_1 is called the difference in "Electric potential" of the points P_2 and P_1.

→ Which point is at higher potential?

The point P_1 is at higher potential as it involves work to be done to reach it from P_2.

(or)

The potential rises from P_2 to P_1.

→ The unit of potential is "Volt"

1 Volt = 1 Joule / Coulomb.

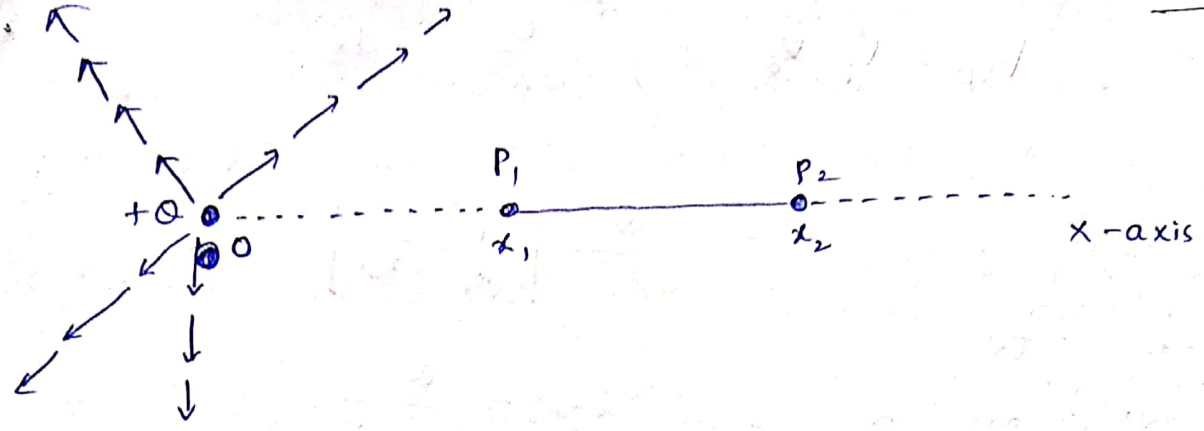

Fig: To find the potential difference b/w P_2 and P_1 due to positive charge Q located at O.

→ The potential is a scalar quantity and is found to be equal to the workdone per unit charge in moving a test charge against the field from a reference point, say, from infinity, to its final position.

→ Consider now a positive charge Q at a point O. This produces a non-uniform, radial field around it.

The workdone per Coulomb of positive test charge from P_2 to P_1 along the x-axis from $x = x_2$ to x_1 equals the potential rise from P_2 to P_1 denoted as V_{21}.

i.e.,
$$V_{21} = \int_{x_2}^{x_1} dv = -\int_{x_2}^{x_1} E \cdot dx$$

The negative symbol signifies that the test charge is moved against the field produced by Q.

$$|E| = \frac{Q}{4\pi\epsilon_0 x^2}$$

$$V_{21} = V_1 - V_2 = -\int_{x_2}^{x_1} \frac{Q}{4\pi\varepsilon_0 x^2} \cdot dx.$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x_1} - \frac{1}{x_2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 x_1} - \frac{Q}{4\pi\varepsilon_0 x_2}$$

where $\left.\begin{aligned} V_1 &= \frac{Q}{4\pi\varepsilon_0 x_1} \\ V_2 &= \frac{Q}{4\pi\varepsilon_0 x_2}\end{aligned}\right\}$

If P_2 is a point at infinitely large distance from O, then

$$\boxed{V_{21} = V_1 = \frac{Q}{4\pi\varepsilon_0 x_1}}$$

If Q is in Coulomb, x_1 in metres, the potential is in volts. This potential of P_1 is called the "absolute potential of the point.

In general, at any point 'p' at a distance 'r' from the point charge 'Q', the absolute potential is given by

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$

The above expression shows that the potential is inversely proportional to the distance.

" Absolute potential is the workdone per Coulomb to bring a positive test-charge from infinity to the point under Consideration".

In other words, if one assumes zero potential at infinity, the potential at a distance r from the point charge is the workdone per unit charge by an external agent in transferring a test charge from infinity to that point. Thus.

$$\boxed{V = -\int_{\infty}^{r} E \cdot dl}$$

→ If a point charge Q is not located at the origin but a point whose position vector is r', the potential $V(x,y,z)$ at r becomes

$$\boxed{V(r) = \frac{Q}{4\pi\varepsilon_0 |r - r'|}}$$

→ In general, For n point charges $Q_1, Q_2 \cdots Q_n$ located at points with position vectors $r_1, r_2, \cdots, r_n$, the potential at r is

$$V(r) = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|} + \frac{Q_2}{4\pi\varepsilon_0 |r - r_2|} + \cdots + \frac{Q_n}{4\pi\varepsilon_0 |r - r_n|}$$

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0} \sum_{K=1}^{n} \frac{Q_K}{|r - r_K|}}$$ (point charges).

→ For the Continuous Charge distributions, we replace Q_k with charge elements $\rho_L \, dl$, $\rho_s \, ds$, $\rho_v \, dv$ and the summation becomes an integration,

The potential at r becomes

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_L \frac{\rho_L(r') \, dl'}{|r-r'|} \quad \text{(line charge)}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\rho_s(r') \, ds'}{|r-r'|} \quad \text{(surface charge)}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_v(r') \, dv'}{|r-r'|} \quad \text{(volume charge)}$$

(P) Two point charges $-4\mu C$ and $5\mu C$ are located at $(2,-1,3)$ and $(0,4,-2)$, respectively. Find the potential at $(1,0,1)$, assuming zero potential at infinity.

Sol: Let $Q_1 = -4\mu C$, $Q_2 = 5\mu C$

$$V(r) = \frac{Q_1}{4\pi\epsilon_0 |r-r_1|} + \frac{Q_2}{4\pi\epsilon_0 |r-r_2|} + C_0$$

If $V(\infty) = 0$, $C_0 = 0$

$$|r-r_1| = |(1,0,1)-(2,-1,3)| = \sqrt{6}$$

$$|r-r_2| = |(1,0,1)-(0,4,-2)| = \sqrt{26}$$

Hence

$$V(1,0,1) = \frac{10^{-6}}{4\pi \times \frac{10^{-9}}{36\pi}} \left[\frac{-4}{\sqrt{6}} + \frac{5}{\sqrt{26}} \right]$$

$$= -5.872 \text{ KV.}$$

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potential ~~at dat~~ at any point due to Charge Q uniformly distributed over the Surface of Sphere, radius R_1:

The absolute potential at a radius $r > R_1$ is given by

$$V = -\int_{\infty}^{r} E \cdot dr$$

$$= \frac{-Q}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{dr}{r^2}$$

$$= \frac{Q}{4\pi\epsilon_0 r} \quad (r \geq R_1)$$

At the Spherical surface ($r = R_1$)

$$V = \frac{Q}{4\pi\epsilon_0 R_1}$$

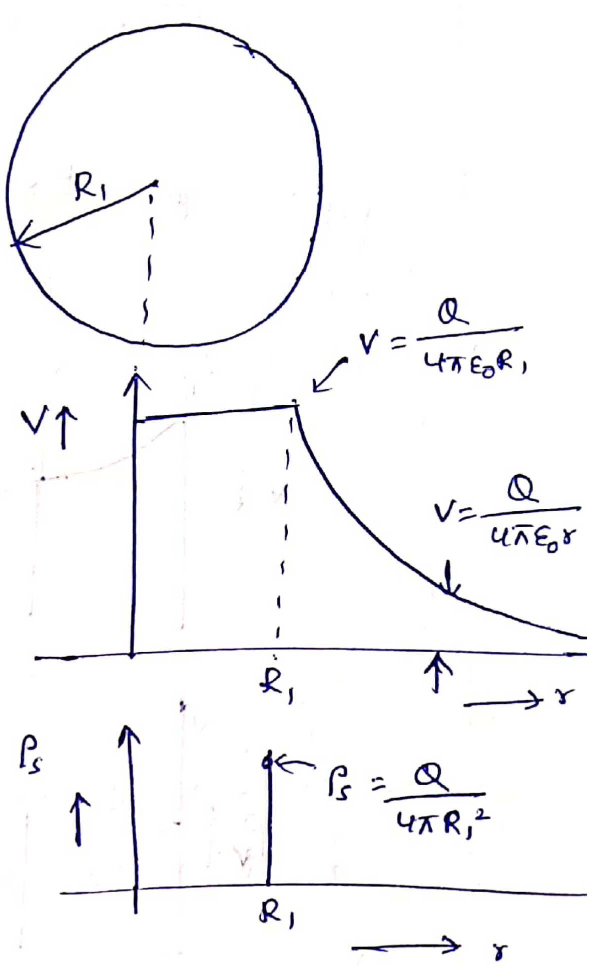

Fig: potential and charge density as function of r.

As E = 0 inside the shell, no work is involved in moving a test charge inside, with the result ~~that~~ the potential remains invariant inside the shell, and will, ~~therefore~~, be the Same as at the Surface.

potential at any point due to the charge Q_t is distributed homogeneously throughout the volume of sphere, radius R :

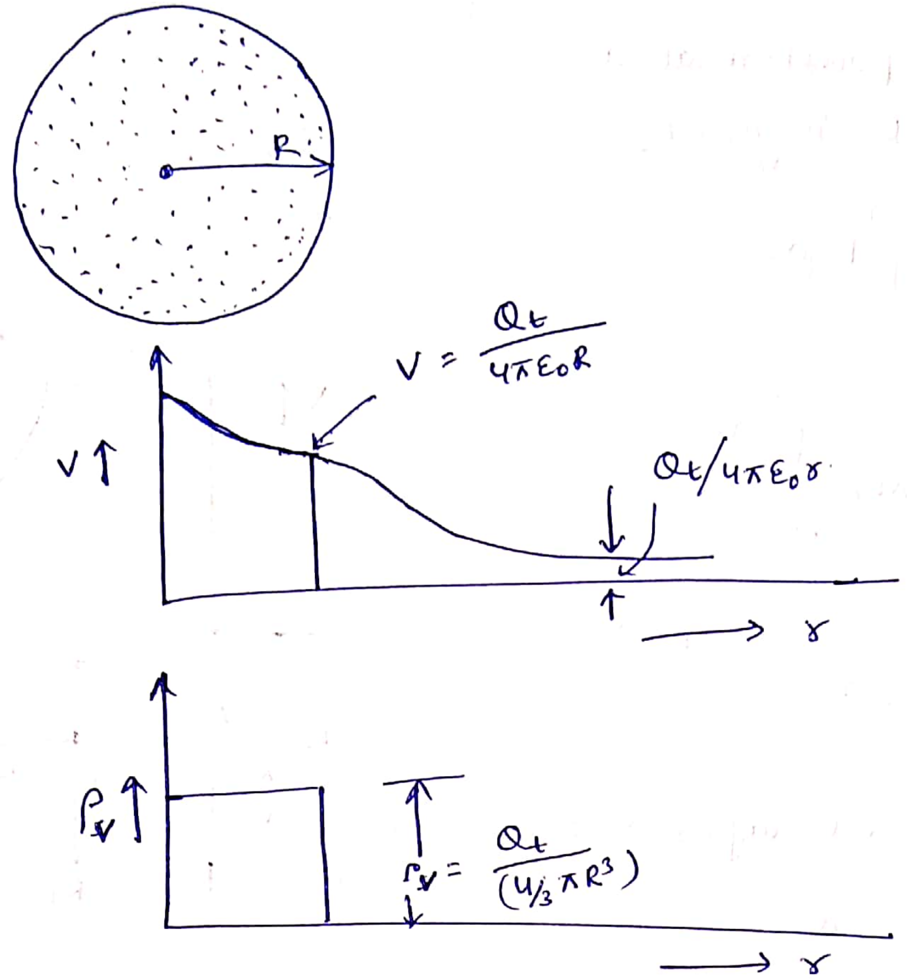

Fig: potential and volume charge density as function of r.

Absolute potential for $r \geq R$

$$V = \frac{Q_t}{4\pi\epsilon_0 r}$$

The potential on the surface of the sphere is

$$V = \frac{Q_t}{4\pi\epsilon_0 R}.$$

The field at any point inside the sphere is

$$E = \frac{Q_t\, r}{4\pi\epsilon_0 R^3}$$

The potential at any point $r \le R$ is

$$V = \frac{Q_t}{4\pi\varepsilon_0 R} - \int_R^r E \cdot dr$$

$$= \frac{Q_t}{4\pi\varepsilon_0 R} - \int_R^r \frac{Q_t r}{4\pi\varepsilon_0 R^3} \cdot dr$$

$$= \frac{Q_t}{4\pi\varepsilon_0 R} - \frac{Q_t}{4\pi\varepsilon_0 R^3} \int_R^r r \cdot dr.$$

$$= \frac{Q_t}{4\pi\varepsilon_0 R} - \frac{Q_t}{4\pi\varepsilon_0 R^3} \left[\frac{r^2}{2}\right]_R^r$$

$$= \frac{Q_t}{4\pi\varepsilon_0 R} - \frac{Q_t}{4\pi\varepsilon_0 R^3} \left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$\boxed{V = \frac{Q_t}{4\pi\varepsilon_0 R} + \frac{Q_t}{8\pi\varepsilon_0 R^3}\left[R^2 - r^2\right]}$$

$$\rho_v = \frac{\text{Total charge}}{\text{Volume of Charge}} = \frac{Q}{\frac{4}{3}\pi R^3}$$

potential due to uniformly charged line :

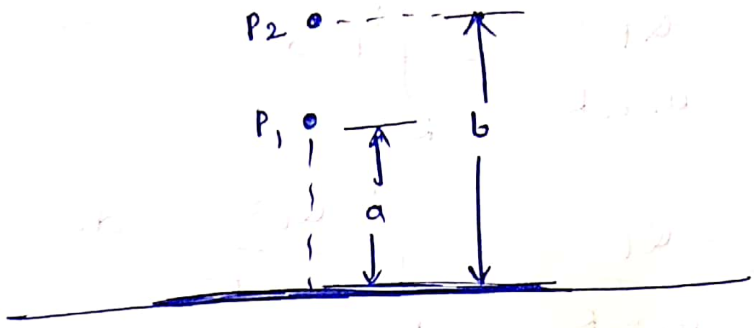

Fig: Infinitely long line, charge density ρ_L.

EFI at any distance r from the line is

$$E = \frac{\rho_L}{2\pi \varepsilon_0 r} \cdot \hat{n}.$$

potential rise from b to a is given by

$$V_{ba} = -\int_b^a E \cdot dr = -\int_b^a \frac{\rho_L}{2\pi \varepsilon_0 r} dr$$

$$= \int_a^b \frac{\rho_L}{2\pi \varepsilon_0 r} dr$$

$$\boxed{V_{ba} = \frac{\rho_L}{2\pi \varepsilon_0} \ln(b/a)}$$

As $b \to \infty$, then expression becomes infinite. i.e., strange condition.

That is why evaluation of potential as an absolute potential is not meaningful. potential difference b/w two points at finite distance is meaningfully representable.

potential and field between two coaxial cylinders

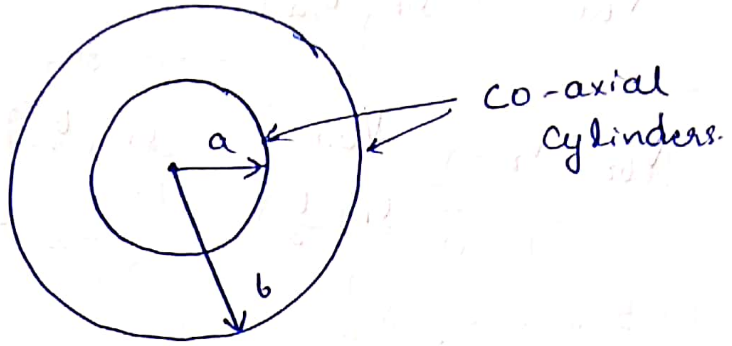

Fig: Cross-sectional View of Co-axial cylinders

By applying Gauss's law, the field at any distance 'r' from the axis of cylinders, for a length l.

$$E\,(2\pi r l) = \frac{\rho_L l}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho_L}{2\pi\varepsilon_0 r}$$

potential rise from r=b to r=a is

$$V_{ba} = -\int_b^a E \cdot dr$$

$$= -\int_b^a \frac{\rho_L}{2\pi\varepsilon_0 r}\, dr$$

$$\boxed{V_{ba} = \frac{\rho_L}{2\pi\varepsilon_0} \ln(b/a)}$$

potential rise from r=b to any value of r b/w a and b

$$\boxed{V_{br} = V_r = \frac{\rho_L}{2\pi\varepsilon_0} \ln(b/r)}$$

$$\frac{V_{br}}{V_{ba}} = \frac{\ln(b/r)}{\ln(b/a)}$$

$$\boxed{V_{br} = V_r = \frac{V_{ba}}{\ln\left(\frac{b}{a}\right)} \ln(b/r)}$$

This gives the p.D b/w any point $(a<r<b)$ and the outer Cylinder, interms of p.D b/w two cylinders.

potential b/w two Conducting spherical shells :

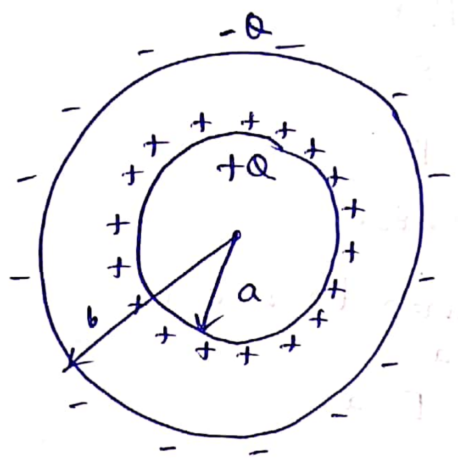

Fig: potential b/w two spherical shells.

EFI at any point b/w 'a' and 'b' is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

potential rise from b to a Can be

$$V_{ba} = -\int_b^a E \cdot dr$$

$$= -\int_b^a \frac{Q}{4\pi\varepsilon_0 r^2} \, dr.$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_b^a$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

Electric Field lines and Equipotential Contours :

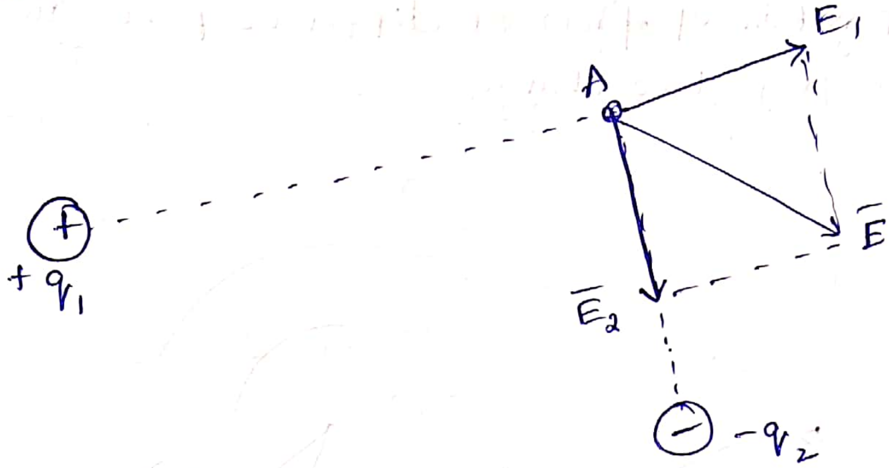

Fig: Electric field produced at A by Combined influence of two charges.

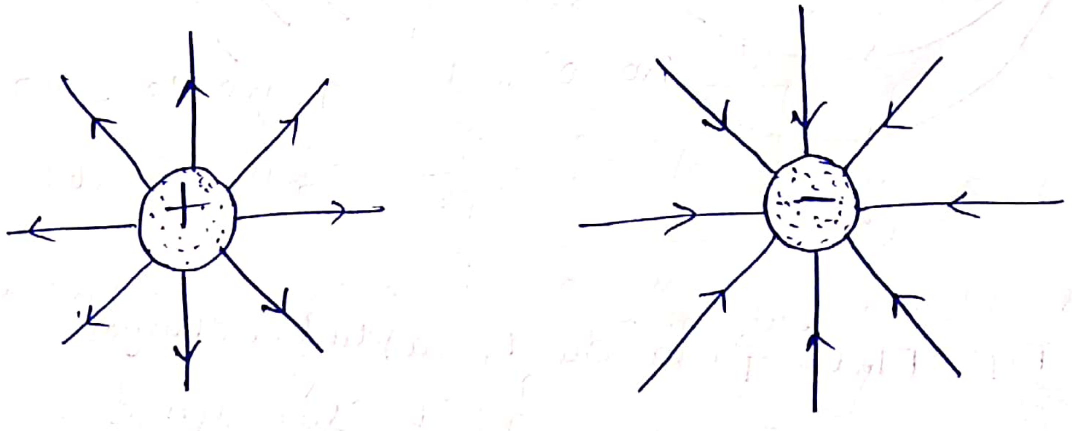

Fig: Lines of force due to Charged sphere.

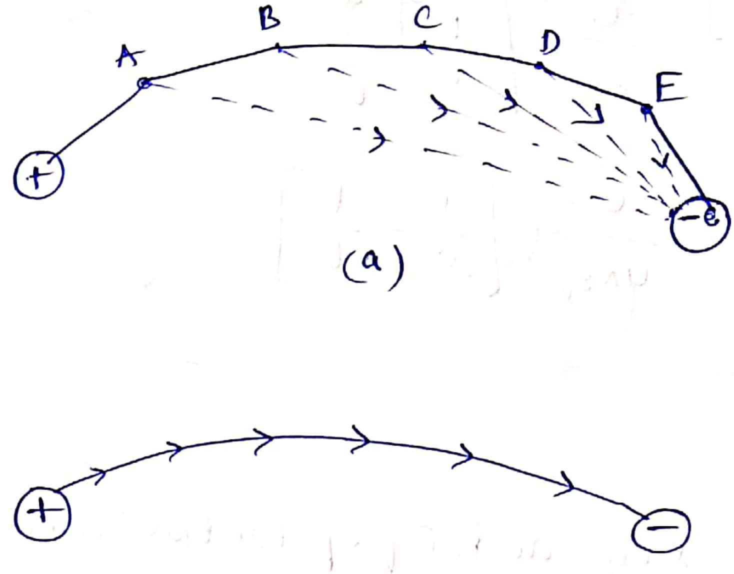

Fig: Direction of field at different points due to two charges.

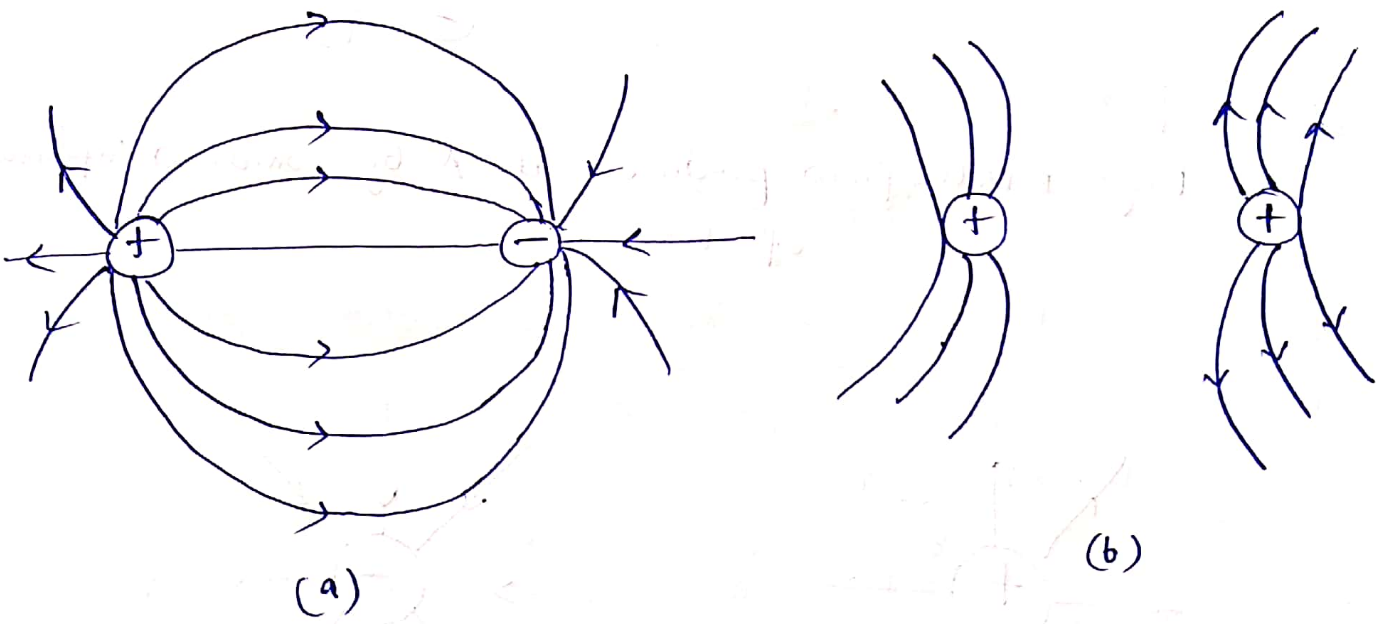

Fig: Electric field due to (a) unlike charges
(b) like charges.

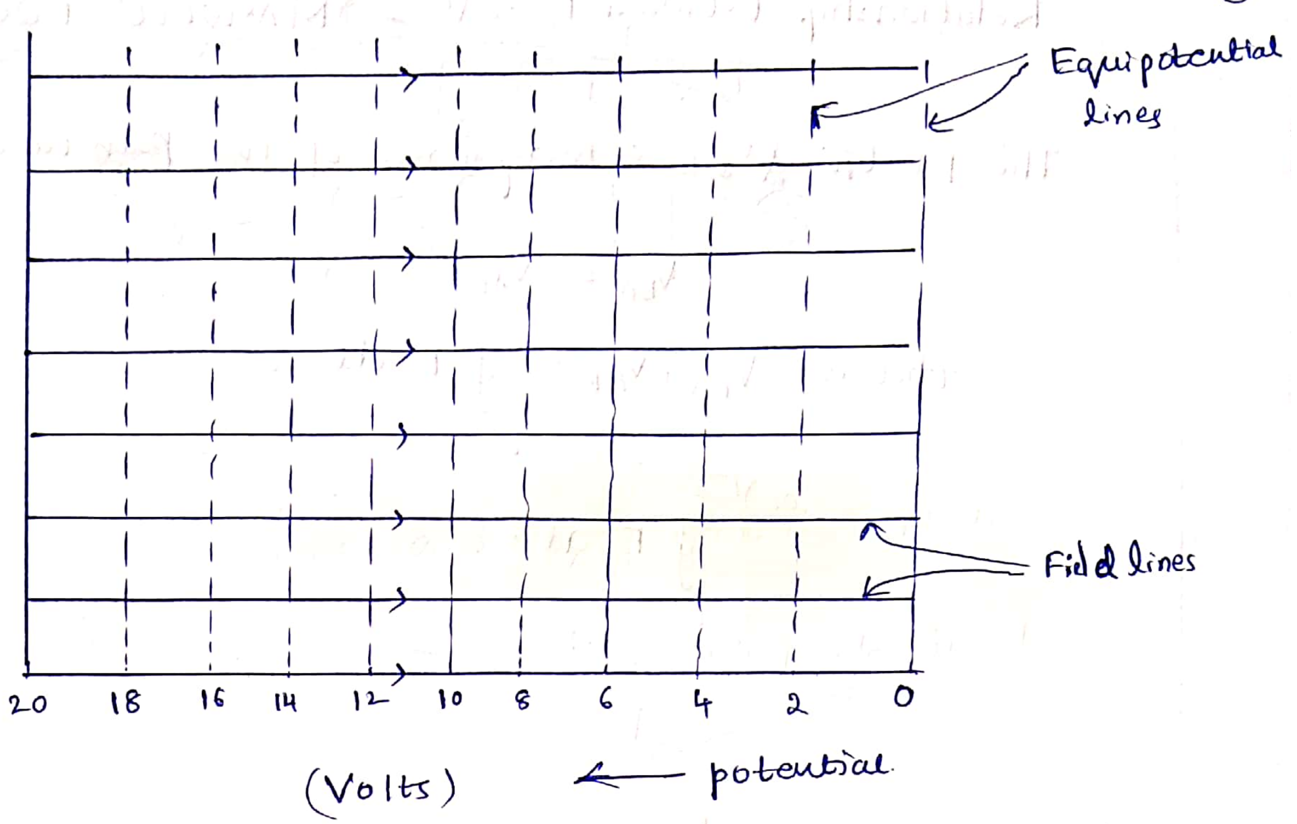

Fig: Uniform electric field lines and equipotential lines.

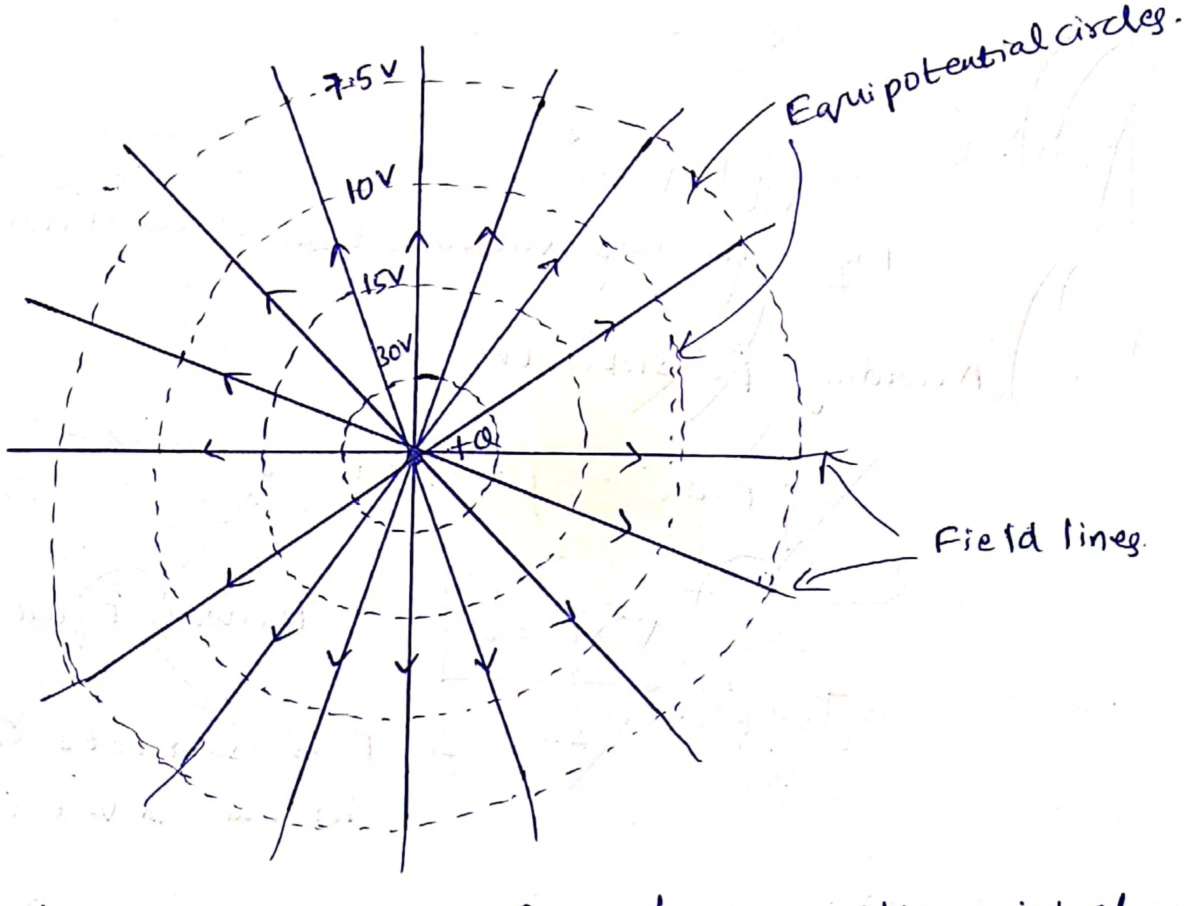

Fig: A non-uniform electric field due to positive point charge.

Relationship between E & V — MAXWELL'S EQUATION:

The p.d b/w A & B is independent of the path taken. Hence

$$V_{BA} = -V_{AB}$$

that is $V_{BA} + V_{AB} = \oint_L E \cdot dl = 0$

(or) $$\boxed{\oint_L E \cdot dl = 0}$$

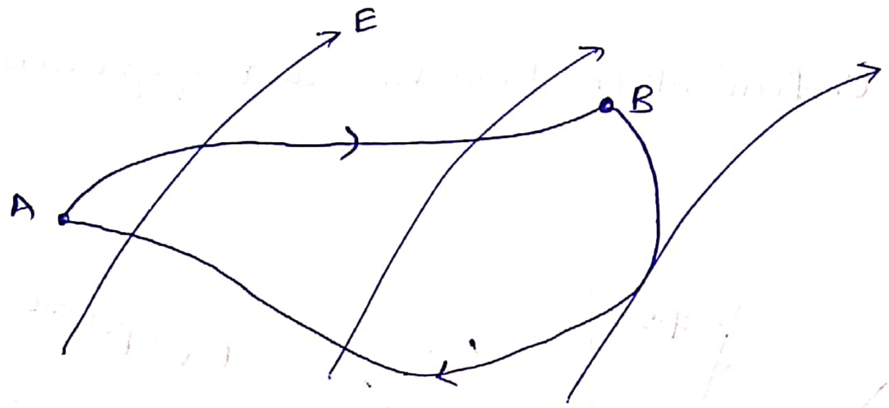

Fig: The Conservative nature of an electostatic field.

According to stokes theorem

$$\oint_L E \cdot dl = \int_S (\nabla \times \vec{E})\, ds = 0$$

$\therefore$ $\boxed{\nabla \times \bar{E} = 0}$ — Maxwell's Equation.

So, $\bar{E}$ is said to be Conservative (or) irrotational Vector.

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$$V_{BA} = V_A - V_B = \int_A^B \bar{E} \cdot dl.$$

Fundamental Theorem of Gradient

$$\int_a^b \nabla \cdot V \cdot dl = V(b) - V(a)$$

$$\int_A^B \nabla V \cdot dl = V_B - V_A.$$

$$-\int_A^B \nabla V \cdot dl = V_A - V_B$$

$$-\int_A^B \nabla V \cdot dl = \int_A^B \bar{E} \cdot dl$$

$$\boxed{\bar{E} = -\nabla V}$$

i.e., EFI is the gradient of V.

The negative sign shows that the direction of $\bar{E}$ is opposite to the direction in V increases.

Poissions & Laplacian equation:

Point form of Gauss's law

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon_0} \rightarrow (1)$$

$$E = -\nabla V \rightarrow (2)$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}}$$ ← poission's Equation.

If $\rho_v = 0$, the poission's equation is

$$\boxed{\nabla^2 V = 0}$$ ← Laplacian equation.

Solution of Laplacian equation in one Variable :

The Laplacian equation is

$$\boxed{\nabla^2 V = 0}$$

Cartesian system $\quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 Z}{\partial z^2} = 0$

V is a function of only 'x'

$$\frac{\partial^2 V}{\partial x^2} = 0$$

$$\frac{\partial V}{\partial x} = A$$

$$\boxed{V = Ax + B}$$

where A & B are Orbitrary Constants

V is a function of only 'y'

$$\frac{\partial^2 V}{\partial y^2} = 0$$

$$\boxed{V = Ay + B}$$

V is a function of only 'z'

$$\frac{\partial^2 V}{\partial z^2} = 0$$

$$\boxed{V = Az + B}$$

Cylindrical form : $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r.\frac{\partial V}{\partial r}\right] + \frac{1}{r^2}.\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

'V' is a function of only 'r'

$$\boxed{V = A\ln r + B}$$

'V' is a function of Only 'φ'

$$\boxed{V = A\phi + B}$$

V is a function of only Z

$$V = AZ + B$$

Spherical form : $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}.\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r^2\sin^2\theta}.\frac{\partial^2 V}{\partial \phi^2} = 0.$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2.\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}.\frac{\partial}{\partial \theta}\left[\sin\theta.\frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}.\frac{\partial^2 V}{\partial \phi^2} = 0$$

'V' is a function of only 'r'

$$\boxed{V = \frac{-A}{r} + B}$$

'V' is a function of only 'θ'

$$V = A\ln\left[\tan\frac{\theta}{2}\right] + B$$

'V' is a function of only 'φ'

$$\boxed{V = A\phi + B}$$

NOTE:

$$\int\frac{1}{\sin\theta}\,d\theta = \int\frac{1}{\sin 2u}\,2.du = \int\frac{1}{\cancel{2}\sin u\cos u}\,\cancel{2}du = \int\frac{\sin^2 u + \cos^2 u}{\sin u\cos u}\,du$$

$\theta = 2u$
$d\theta = 2.du$

$$= -\int\frac{-\sin u}{\cos u}\,du + \int\frac{\cos u}{\sin u}\,du$$

$t = \cos u$, $dt = -\sin u\,du$; $v = \sin u$, $dv = \cos u\,du$

$$= -\int\frac{1}{t}\,dt + \int\frac{1}{v}\,dv = -\ln|t| + \ln|v| = \ln\left|\frac{v}{t}\right|$$

$$= \ln\left|\frac{\sin u}{\cos u}\right| = \ln|\tan u| = \ln\left|\tan\frac{\theta}{2}\right| + B.$$

$$\nabla V = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right] V$$ Cartesian

$$\nabla V = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}$$

$$\nabla \cdot \nabla V = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\nabla^2 V = 0$$

Cylindrical.

$$\nabla \cdot V = \left[\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{k}\right] V$$

$$\nabla \cdot V = \left[\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\nabla \cdot \nabla \cdot V = \left[\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\cdot\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{k}\right]\left[\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$= \left[\frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$= \frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r\cdot\frac{\partial V}{\partial r}\right] + \frac{1}{r^2}\cdot\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

Spherical

$$\nabla \cdot V = \left[\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\cdot\hat{\phi}\right] V$$

$$= \left[\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\cdot\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\cdot\hat{\phi}\right]$$

$$\nabla \cdot \nabla V = \left[\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{\phi}\right]$$
$$\left[\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\cdot\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\cdot\hat{\phi}\right]$$

$$= \left[\frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\cdot\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$V = Ax + B$, $V = Ay + B$, $V = Az + B$

$\frac{1}{r}\frac{\partial}{\partial r}\left[r \cdot \frac{\partial V}{\partial r}\right] = 0$

$r \cdot \frac{\partial V}{\partial r} = A$

$\frac{\partial V}{\partial r} = \frac{A}{r}$

$V = A \ln r + B$

$\frac{1}{r^2} \cdot \frac{\partial^2 V}{\partial \phi^2} = 0$

$\Rightarrow \frac{\partial^2 V}{\partial \phi^2} = 0$

$V = A\phi + B$

$V = Az + B$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \cdot \frac{\partial V}{\partial r}\right] = 0$

$r^2 \cdot \frac{\partial V}{\partial r} = A$

$\frac{\partial V}{\partial r} = \frac{A}{r^2}$

$V = \frac{-A}{r} + B$

$\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta \cdot \frac{\partial V}{\partial \theta}\right] = 0$

$\sin\theta \cdot \frac{\partial V}{\partial \theta} = A$

$\frac{\partial V}{\partial \theta} = A \operatorname{cosec}\theta$

$\int \frac{1}{\sin x} dx = \int \frac{1}{\sin 2u} 2\,du$

$x = 2u$

$dx = 2du$

$= \int \frac{1}{2 \sin u \cos u} 2\,du$

$= \int \frac{\sin^2 u + \cos^2 u}{\sin u \cos u} du$

$= -\int \frac{-\sin u \, du}{\cos u} + \int \frac{\cos u}{\sin u} du$

$t = \cos u$

$dt = -\sin u \, du$

$v = \sin(u)$

$dv = \cos u \, du$

$= -\int \frac{1}{t} dt + \int \frac{1 \, dv}{v}$

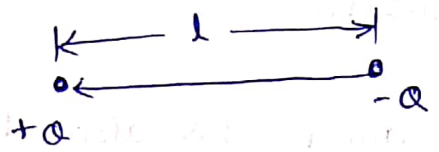
$= -\ln|t| + \ln|v|$

$= \ln|v| - \ln|t| = \ln\left|\frac{v}{t}\right| = \ln\left|\frac{\sin(u)}{\cos(u)}\right|$

$= \ln|\tan(u)| = \ln\left|\tan\frac{\theta}{2}\right| + B$

Electric Dipole:

→ Two point charges +Q and −Q, of equal magnitude and Opposite direction are seperated by a small distance 'l' called "electric dipole".



→ The product of charge and spacing, Ql, is referred to as the "Electric Dipole moment".

$$\boxed{\bar{m} = Q\bar{l}}$$

where $\bar{l} = l\, u_e$

m = electric dipole moment.

u_e = Unit Vector directed along the dipole axis.

(a) Potential at "p" due to dipole.

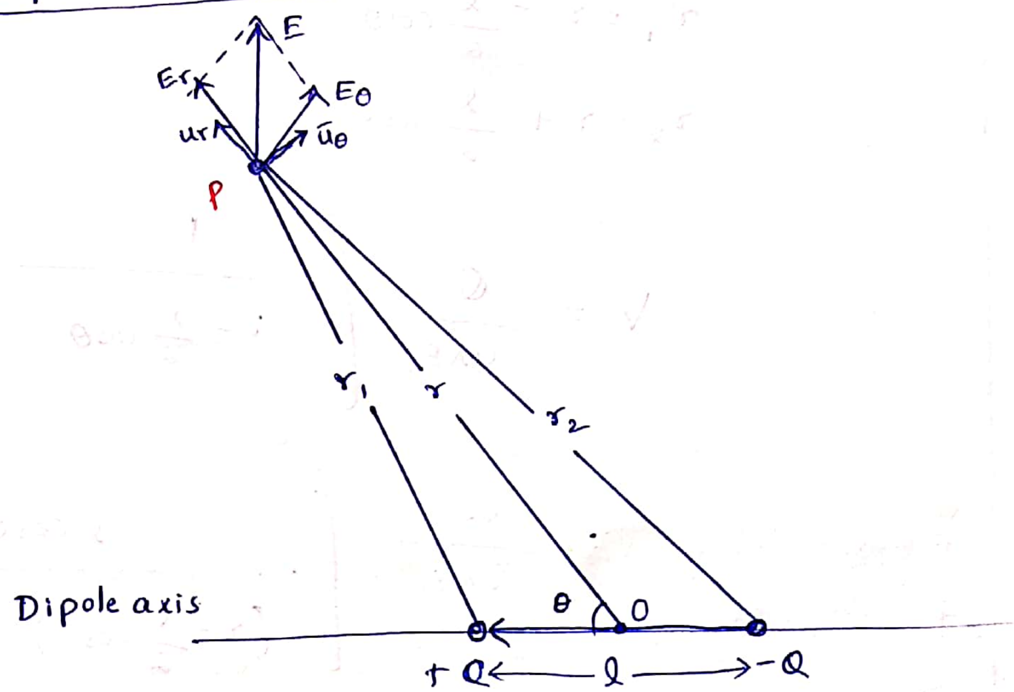


Fig: Dipole: Charge-configuration formed by equal and Opposite charges closely spaced.

→ Let p be at distances of r_1, r_2 and r respectively from +Q, −Q and from the midpoint O of the line joining the charges.

→ The potential at p will be zero if the charges are Superimposed to each other. As they are seperated and $r_1 \neq r_2$, the potential will be non-zero.

The potential at p due to +Q alone is

$$V_1 = \frac{Q}{4\pi\epsilon_0 r_1}$$

and that due −Q alone is

$$V_2 = \frac{-Q}{4\pi\epsilon_0 r_2}$$

The resultant potential at p is thus

$$V = V_1 + V_2 = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

From the diagram

$$r_1 = r - \frac{l}{2}\cos\theta$$

$$r_2 = r + \frac{l}{2}\cos\theta.$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r - \frac{l}{2}\cos\theta} - \frac{1}{r + \frac{l}{2}\cos\theta}\right]$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{l\cos\theta}{\left(r - \frac{l}{2}\cos\theta\right)\left(r + \frac{l}{2}\cos\theta\right)}\right]$$

$$\simeq \frac{Q}{4\pi\epsilon_0}\frac{l\cos\theta}{r^2}$$

(as $r >>> l$, denominator $\longrightarrow r^2$)

The potential at any point p is approximately given by

$$V = \frac{Ql\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\boxed{V = \frac{m\cos\theta}{4\pi\epsilon_0 r^2}}$$

→ This clearly indicates that potential along the perpendicular bisector to dipole-axis is zero (as $\theta = 90°$).

→ The expression also indicates that the potential increases proportionately with the dipole moment and inversely with the square of the distance.

(b) Electric field produced at P due to dipole :

As $V = -\int E \cdot dr$ can be used for evaluation of field at P due to +Q and −Q seperately.

→ The field intensity E has components along the radial line OP and in the direction of increasing θ.

Let these components be E_r and E_θ respectively.

$$\vec{E} = E_r + E_\theta$$

$$= -u_r \cdot \frac{\partial V}{\partial r} - u_\theta \cdot \frac{\partial V}{r\partial \theta}$$

where u_r = unit vector in the direction of increasing value of r, directed along OP.

u_θ = Unit Vector in the direction (perpendicular to OP) of increasing value of θ.

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$$\frac{\partial V}{\partial r} = \frac{-m\cos\theta}{2\pi\varepsilon_0 r^3}$$

$\left[\frac{d}{dr}(r^{-2}) = -2 \cdot r^{-2-1} = \frac{-2}{r^3}\right]$

$$\frac{\partial v}{\partial \theta} = \frac{-m\sin\theta}{4\pi\varepsilon_0 r^2}$$

$$E = u_r\left[\frac{m\cos\theta}{2\pi\varepsilon_0 r^3}\right] + u_\theta\left[\frac{m\sin\theta}{4\pi\varepsilon_0 r^3}\right]$$

where $E_r = \dfrac{m\cos\theta}{2\pi\varepsilon_0 r^3}$

$$E_\theta = \frac{m\sin\theta}{4\pi\varepsilon_0 r^3}$$

NOTE: (1) If $\theta = 90^\circ$, $E_r = 0$ and E_θ persists.

(2) If $\theta = 0$, obviously p is some where in alignment with dipole axis & E_θ becomes zero.

(c) Torque experienced by a Dipole in a uniform field:

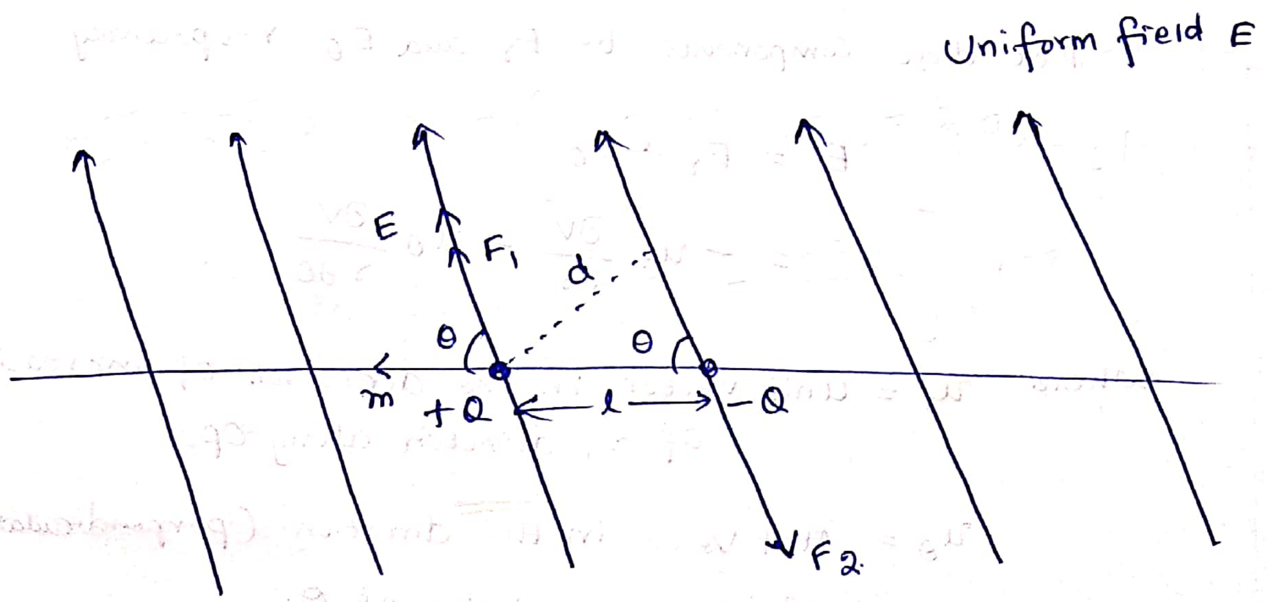

Fig: Torque on a dipole.

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→ We have seen earlier that $m = Ql$ = dipole moment is a Vector directed from negative to positive charge forming the dipole.

→ When a dipole is placed in a uniform field, the dipole experiences no translational force, as the forces ~~are~~ F_1 and F_2 neutralize each other; but these forces form a Couple whose torque is equal in magnitude to force × arm of Couple

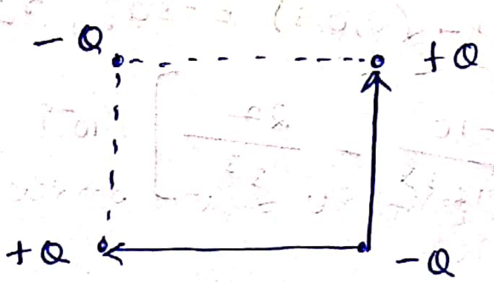
$$T = (QE)\,d$$
$$= (QE)\,(l\sin\theta)$$
$$= Ql\,E\sin\theta$$
$$= mE\sin\theta$$

and can be represented in the ~~form~~ Vector form by

$$\boxed{\begin{aligned} T &= m \times E \\ |T| &= mE\sin\theta \end{aligned}}$$

→ In Conclusion, although a dipole in a uniform field does not experience a translational force, it does experience a torque tending align the dipole axis with the field.

(d) Quadrupole:



→ Four charges of alternating signs situated at the vertices of a parallelogram form known as "Quadrupole".

Fig: Quadrupole

OBs potential at a point 'p' due to dipole

$$V = \frac{Q\, l \cos\theta}{4\pi\varepsilon_0 r^2}$$

Since $l\cos\theta = l\cdot \bar{a}_r$, where $l = l\, u_e$

$$m = Ql.$$

$$\therefore \quad V = \frac{m\cdot a_r}{4\pi\varepsilon_0 r^2}$$

Note that the dipole moment m is directed from $-Q$ to $+Q$.

If the dipole center is not at the origin but at r'

$$\boxed{V(r) = \frac{m\cdot(r-r')}{4\pi\varepsilon_0 |r-r'|^3}}$$

(P) Two dipoles with dipole moments $-5a_z$ nC·m and $9a_z$ nC·m are located at points $(0,0,-2)$ and $(0,0,3)$, respectively. Find the potential at the origin.

Sol

$$V = \sum_{K=1}^{2} \frac{m_K\cdot r_K}{4\pi\varepsilon_0 r_K^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{m_1\cdot r_1}{r_1^3} + \frac{m_2 r_2}{r_2^3}\right]$$

$m_1 = -5a_z$, $r_1 = (0,0,0)-(0,0,-2) = 2a_z$, $r_1 = |r_1| = 2$

$m_2 = 9a_z$, $r_2 = (0,0,0)-(0,0,3) = -3a_z$, $r_2 = |r_2| = 3$

$$V = \frac{1}{4\pi\frac{10^{-9}}{36\pi}}\left[\frac{-10}{2^3} - \frac{27}{3^3}\right]\cdot 10^{-9}$$

$$= -20.25\ V.$$

Conductor and Dielectric :

→ From the electrical point of view, all substances may be classified in accordance with the freedom or ease with which electric charges can move in them.

→ In broad sense, materials can be classified in terms of their conductivity σ, in mhos per meter ($\mho$/m) or Siemens/meter. The conductivity depends on temperature and frequency.

→ Conductor is one in which the outer electrons of an atom are easily detachable and will migrate with the application of even a weak electric field.

(or)

A material with high conductivity ($\sigma >> 1$) is referred to as a metal or conductor.

→ An insulator or dielectric is one in which the electrons are rigidly bound to their nucleus so that ordinary field will not be able to detach them. However a dielectric placed in an electrostatic field will be subjected to electrostatic induction.

(or)

A material with low conductivity ($\sigma << 1$) is referred to as insulator.

→ There is a fundamental difference between the electrostatic induction of a conductor and a dielectric. As said above, the application of an electric field causes the free electrons to move, through the entire volume of the material, Whereas, in a dielectric, the charged elements are unable to move. The electric field will twist and strain the molecules to orient the positive charges in the direction of the field and the negative charges oppositely. This condition is called "polarization"

→ Generally speaking, polarization of a dielectric is an elastic shift of electric elements in the dielectric material.

If the electric field strength is too great, the dielectric will breakdown i.e., it will cease to be an insulator.

→ A material whose conductivity lies somewhere between those of conductors and insulators is called a "Semi conductor"

→ Alluminium & Copper are good examples of Conductors

Glass & Rubber are good examples of insulators

Silicon & Germanium are good examples of Semiconductors.

KNLchary (15)

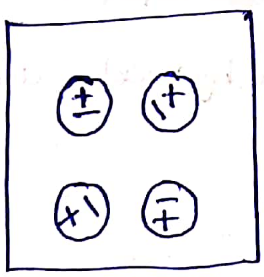
polarization

→ There are two classes of dielectric materials :

In the first category, the positive and negative elements in the uncharged condition are so close to each other that their action is neutralised. The application of an electric field will shift the positive and negative charges slightly with in the molecules to give rise to a 'Dipole'.

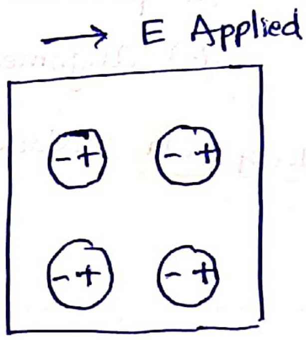
In the other category, the molecules form dipoles even in the absence of an electric field. Such dielectrics (eg., water, ammonia, ether, acetone, etc) are called polar dielectrics. In the absence of field, however, the dipoles are disposed at random : The resultant electric field of polar dielectric is zero.

On the application of field, the dipoles rearrange themselves so that their axes are in alignment with the applied field. This shift results in instantaneous Current, Called "Displacement current" which ceases in a very small fraction of second.



Fig(1)

polar dielectric without applied electric field



Fig(2)

polar dielectric with applied electric field

→ Consider a dielectric material cut in the form of a slab of permitivity 'ϵ' in fig (3) and situated in a uniform applied field E_a in a direction normal to slab.

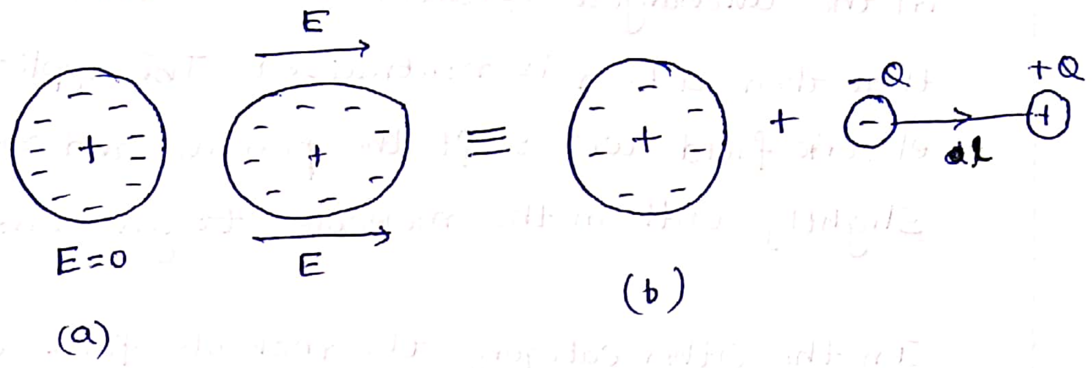


Fig: polarization of a nonpolar atom or molecule.

Examples of Non-polar: hydrogen, oxygen, nitrogen and raregases

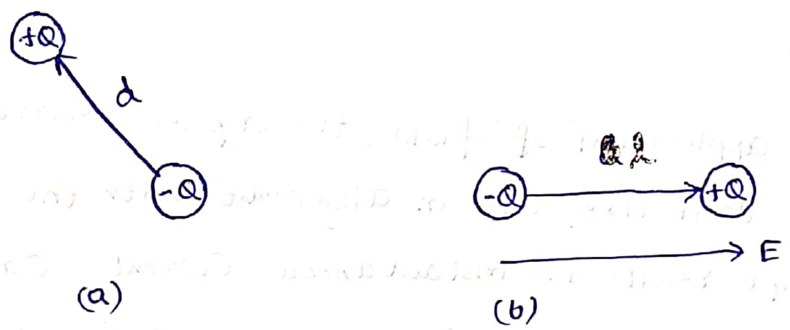


Fig: polarization of a polar molecule:
(a) permanent dipole ($E=0$)
(b) alignment of permanent dipole ($E \neq 0$)

Examples of polar: Water, Sulpher dioxide, hydrochloric acid & polystyrene.

→ The effect of the field is to polarize the dielectric, inducing atomic dipoles throughout the volume of the specimen in alignment with the field.

→ Consequent to neutralization of equal and opposite charges inside the dielectric, ultimately charges reside on the left and right surfaces of the slab; $-Q$ and $+Q$ respectively representing the corresponding charges (seperated through a distance l).

→ The product Ql is called the net dipole moment in volume (Al), where A is the surface area of the slab.

→ Dipole moment per unit volume is called "polarization", a vector directed from $-Q$ to $+Q$.

$$\boxed{P = \frac{Ql}{Al} = \frac{Q}{A}\, u_l} \longrightarrow (1)$$

where u_l is a unit vector directed from $-Q$ to $+Q$.

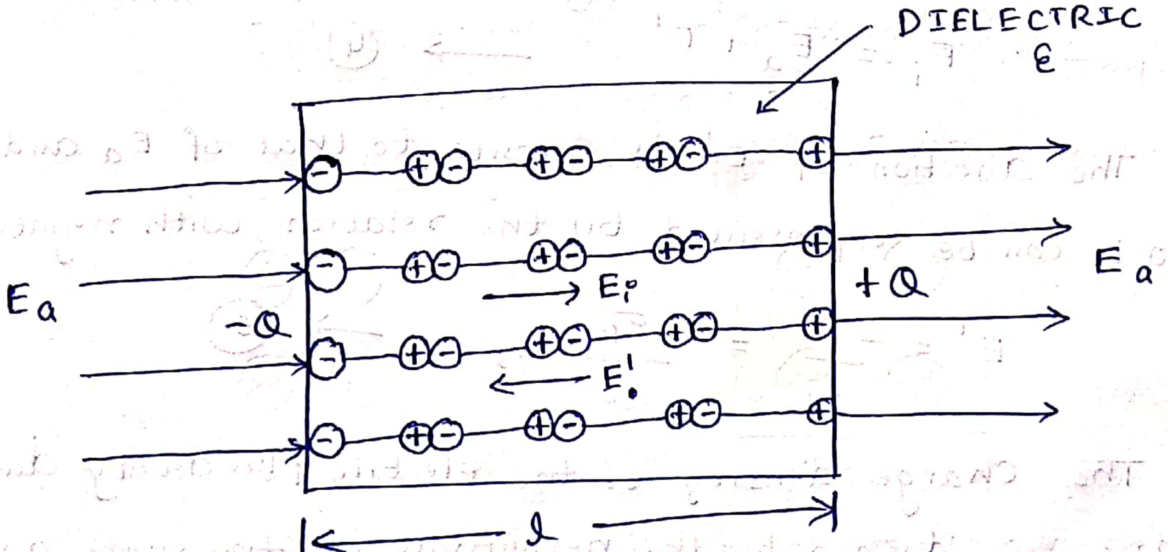


Fig: Dielectric situated in a uniform field in a vaccum.

→ Thus the polarization has a dimension of dipolemoment per unit volume and also of charge per unit area i.e., Surface charge density, σ_P.

σ_P = Surface charge density of polarisation.

Thus

$$P = \frac{Q}{A} = \sigma_P \longrightarrow (2)$$

or

$$P = \lim_{\Delta V \to 0} \frac{m}{\Delta V} \longrightarrow (3)$$

→ In a free space region, outside the slab, field intensity is E_a, However, due to the induced surface charge σ_P due to polarization, a field E' is induced in the slab opposing the applied field, with the result that, field inside the slab is the vector sum of the applied field and the induced field.

The internal field is

$$E_i = E_a + E' \longrightarrow (4)$$

The direction of E' is opposite to ~~that of~~ E_a and so it can be represented by the relation, with negative sign, as

$$E' = -\frac{P}{\varepsilon_0} = -\frac{\sigma_P}{\varepsilon_0} u_x \longrightarrow (5)$$

The charge density or the electric flux density due to polarization divided by the permitivity of free space gives the field induced due to polarization. Thus

$$|P| = \sigma_P = -\varepsilon_0 E' \longrightarrow (6)$$

→ It is clear that any positive test charge located inside the dielectric slab will experience an attractive force towards $-Q$ on the L.H.S and a repulsive from $+Q$ on R.H.S.

The direction of force is thus from right to left and this is the direction of E' which is obviously opposite to that of the applied field.

→ The resultant field in the dielectric is reduced to E_i.

→ In most of the materials, the change in field due to polarization is proportional to E_i.

$$\boxed{\sigma_p = P = \chi E_i} \longrightarrow (7)$$

where χ is a proportionaliy constant and a scalar quantity for isotropic materials.

P and E_i are then in same direction

The above constant $\chi = \frac{P}{E_i}$ is called dielectric suceptibility. dipicting the ratio of polarisation to internal field intensity.

From Eq (4)

$$E_i = E_a + E'$$

$$E_i = E_a + \left(-\frac{P}{\varepsilon_0}\right)$$

$$E_i = E_a - \frac{\chi E_i}{\varepsilon_0}$$

$$E_a = E_i\left[1 + \frac{\chi}{\varepsilon_0}\right]$$

$$\boxed{E_i = \frac{E_a}{\left[1 + \frac{\chi}{\varepsilon_0}\right]}} \longrightarrow (8)$$

→ If the dielectric is replaced in free space, E_i would be the applied field itself i.e.,

$$E_i = E_a$$

However, if the dielectric is re-inserted, E_i is reduced, as a result of polarization to $\dfrac{E_a}{\left[1+\frac{\chi}{\varepsilon_0}\right]}$.

→ Let $1+\dfrac{\chi}{\varepsilon_0} = \varepsilon_r$

the internal field

$$E_i = \frac{E_a}{\varepsilon_r}.$$

Where ε_r is known as the relative-permitivity of the dielectric. This is readily acceptable as, for free space,

$$E_i = E_a$$

So that, $\varepsilon_r = 1$

with which Comparision is made for any dielectric.

→ From (8), $E_a = E_i\left[1+\dfrac{\chi}{\varepsilon_0}\right]$

Multiplying with ε_0 on both sides

$$\varepsilon_0 E_a = \varepsilon_0 E_i + \chi E_i \longrightarrow (9)$$

= Electric flux density in the dielectric (normal to the slab surface)

If $\bar{D}$ = normal component of the electric flux density.

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$$\bar{D} = \varepsilon_0 E_i + P \longrightarrow (10)$$

$$\bar{D} = \left[\varepsilon_0 + \frac{P}{E_i}\right] E_i \longrightarrow (11)$$

$$\bar{D} = \varepsilon E_i \longrightarrow (12)$$

where ε = Absolute permitivity of the dielectric

$$= \varepsilon_0 + \frac{P}{E_i}$$

$$= \varepsilon_0 + \chi$$

$$= \varepsilon_0 \left[1 + \frac{\chi}{\varepsilon_0}\right]$$

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r} \longrightarrow (13)$$

→ The polarization interms of the applied field and the internal fields.

Thus $$\boxed{P = \varepsilon_0 [E_a - E_i]} \longrightarrow (14)$$

$$E_i = E_a + E'$$

$$E_i = E_a - \frac{P}{\varepsilon_0}$$

$$\Rightarrow E_i \varepsilon_0 = \varepsilon_0 E_a - P$$

$$\boxed{P = \varepsilon_0 [E_a - E_i]}$$

The polarization equals the difference between the applied and internal fields multiplied by ε_0.

(P) A dielectric slab of flat surface with $\varepsilon_r = 4$ is disposed with its surface normal to a uniform field with flux density $1.5\ C/m^2$. The slab occupies a volume of 0.08 cubic meter and is uniformly polarised. Determine

(a) the polarisation in the slab, and

(b) The total dipole-moment of slab.

Sol: (a) From Equations

$$D = \varepsilon E$$

$$= \varepsilon_0 \varepsilon_r E$$

$$= 1.5\ C/m^2$$

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{1.5}{4\varepsilon_0} \quad (\text{as } \varepsilon_r = 4)$$

$$\varepsilon_0 E = \frac{1.5}{4}.$$

From relation, $P = D - \varepsilon_0 E = \varepsilon_0 \varepsilon_r E - \varepsilon_0 E$

$$= 1.5 - \frac{1.5}{4}$$

$$= 1.125\ C/m^2$$

(b) The total dipole moment $= P \times \text{Volume}$

$$= 1.125 \times 0.08$$

$$= 0.09\ C\text{-}m.$$

OBSERVATIONS

(i) Unpolarized Atom:

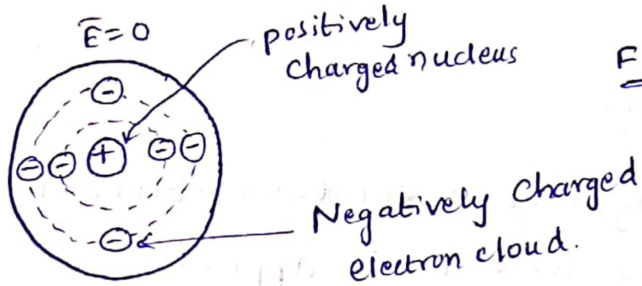


Fig: Unpolarized Atom of a Dielectric.

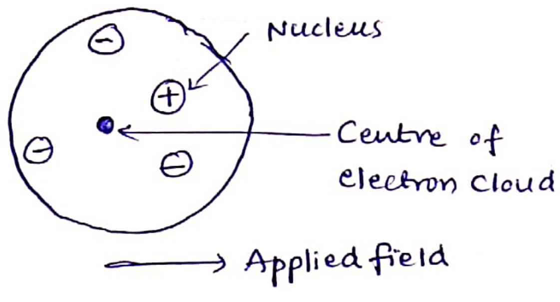
→ when $\bar{E}$ is not applied to an atom of dielectric, number of positive charges is same as the negative charges and hence the atom is electrically neutral.

→ positively charged necleus is present at the center and negatively charged electrons are revolving around the necleus in orbits.

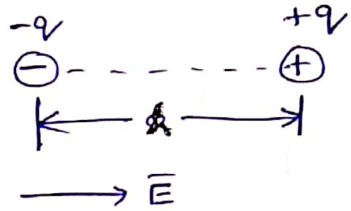
→ All the negatively charged electrons are in the form of electron cloud.

(ii) POLARIZED ATOM:

→



Equivalent dipole



→ When $\bar{E}$ is applied, symmetrical distribution of charges gets disturbed.

→ +ve charges experience a force $\bar{F} = Q\bar{E}$
-ve charges experience a force $\bar{F} = -Q\bar{E}$

→ Now the atom is called polarized atom, in which there is the seperation between the necleus and the centre of electron cloud.

→ polarization of dielectrics is the process in which the dipole gets aligned with the applied field.

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Types of Dielectrics

(1) Non polar Dielectric

(2) Polar Dielectric

Non polar Dielectric :

Dipole arrangement is ~~ab~~totally absent in the absence of $\bar{E}$. It results only when $\bar{E}$ is applied.

Ex: Hydrogen, Oxygen and Rare gases.

polar Dielectric :

Dipole arrangement exists without application of $\bar{E}$, but with random ~~arrangement~~ Orientation.

Under the application of $\bar{E}$, dipole align with the direction of $\bar{E}$.

Ex: Water, Sulphur dioxide etc.

Current and current density :

Current : Current is defined as the rate of flow of charge.

Units : Amperes (A).

Current Density : It is defined as the current passing through the unit Surface area ~~at~~ when the Surface is held normal to the direction of the Current.

Units : Ampere/square metres (A/m^2)

Current

1. Drift Current (or) Conduction Current
2. Displacement Current or Convection ~~ve~~ current.

Drift Current : This current exists in Conductors due to the drifting of electrons under the influence of the applied Voltage.

Displacement Current : This Current exists in dielectrics, due to the flow of charges under the influence of electric field ($\bar{E}$).

Eg: Current flowing through the Capacitor.

Conventional current : Current flow from +ve terminal to the -ve terminal.

Electron flow Current : Current flow from -ve terminal to +ve terminal.

Convection and Conduction Currents :

"The current (in amperes) through a given area is the electric charge passing through the area per unit time."

$$\boxed{I = \frac{dQ}{dt}} \longrightarrow (1)$$

"The current density at any point is the current through a unit normal area at that point."

$$J = \frac{\Delta I}{\Delta S}$$

$$\Delta I = J \cdot \Delta S$$

Thus, the total current flowing through a surface S is

$$\boxed{I = \int_S J \cdot dS} \longrightarrow (2)$$

Depending on how I is produced, there different kinds of current density : Convection current density, Conduction current density and displacement current density.

Case (A): CONVECTION CURRENT :

Convection current, as ~~distant~~ distinct from Conduction current, does not involve conductors and consequently does not satisfy Ohm's law. It occurs, when a current flows through an insulating medium such as liquid, rarefied gas, or a Vaccum.

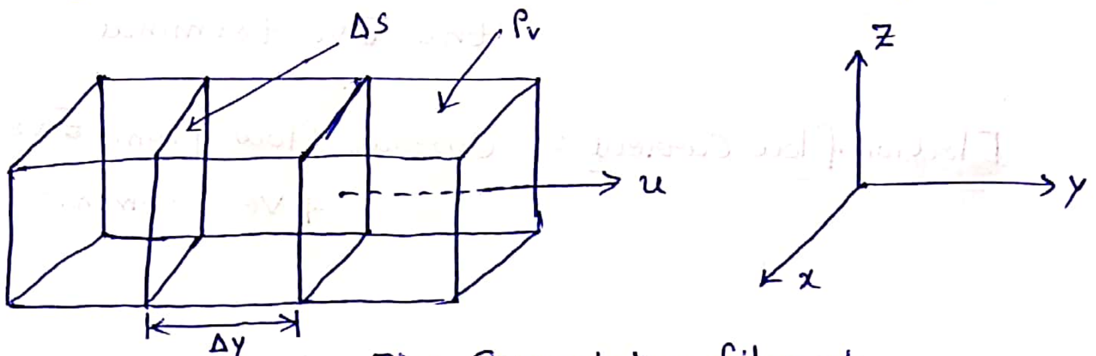


Fig: Current in a filament.

Consider a filament shown in figure. If there is a flow of charge, of density ρ_v, at velocity $u = u_y a_y$.

The current through the filament is

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \cdot \Delta S \frac{\Delta y}{\Delta t} = \rho_v \cdot \Delta S \cdot u_y \longrightarrow (3)$$

The y-directed current density J_y is given by

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v \cdot u_y \longrightarrow (4)$$

Hence in general

$$\boxed{J = \rho u} \longrightarrow (5)$$

The current I is the convection current and J is the convection current density in amperes/square meter (A/m^2)

Case(B): ~~Cond~~ CONDUCTION CURRENT:

→ Conduction current requires a conductor. A conductor is characterized by a large number of free electrons that provide conduction current due to an impressed electric field.

→ When an electric field E is applied, the force on an electron with charge $-e$ is

$$\boxed{F = -eE} \longrightarrow (6)$$

Since the electron is not in free space, it will not experience an average acceleration under the influence of the electric field. Rather it suffers constant collisions with the atomic lattice and drifts from one atom to another. This is called "drifting of electrons"

If an ~~atom~~ electron with mass 'm' is moving in an electric field $\bar{E}$ with an average drift velocity u, according to Newton's law, the average change in momentum of the free electron must match the applied force. Thus,

$$\frac{mu}{\tau} = -eE$$

$$\boxed{u = -\frac{e\tau}{m}E} \Rightarrow \boxed{u = -\mu_e \bar{E}}$$

where τ is the average time interval between collisions.

- Drift Velocity: After sometime, the electrons attain the constant average velocity called "drift Velocity"

Drift Current: The current constituted due to the drifting of electrons in metallic conductors is called drift current.

Mobility: The drift velocity is directly proportional to the applied electric field.

$$u \propto \bar{E}$$

$$\boxed{u = -\mu_e \bar{E}}$$

→ μ_e denote constant of proportionality is called mobility of the electrons.

→ $-$ve sign indicates that the velocity of electrons is against the direction of electric field $\bar{E}$.

$$\boxed{\mu_e = \frac{\text{velocity}}{\text{field}}} \quad \frac{m/s}{V/m} \text{ (or) } \left(\frac{m^2}{V\text{-}s}\right)$$

If there are n electrons per unit volume, the electron charge density is given by

$$\rho_v = -ne$$

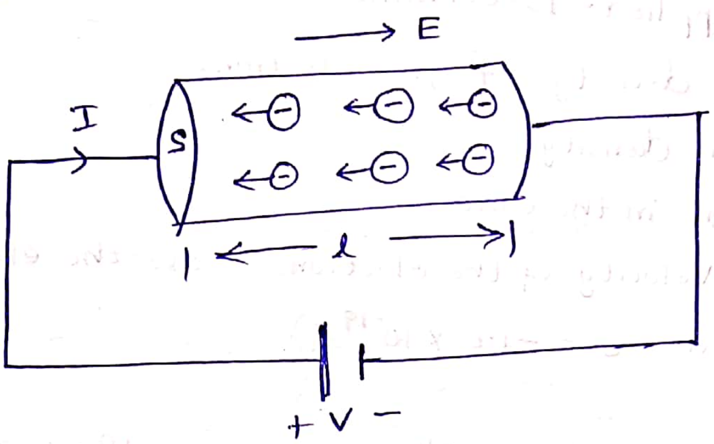
Thus the Conduction Current density is

$$J = \rho_v u = \frac{ne^2 \tau}{m} \cdot E = \sigma E$$

$$\boxed{J = \sigma E}$$ ← point form of Ohm's Law.

where $\sigma = \frac{ne^2\tau}{m}$ is the Conductivity of the Conductor.

⇒



→ The direction of $\bar{E}$ is same as the direction of the flow of positive charges or current I.

→ This direction is opposite to the direction of flow of electrons.

Fig: A Conductor of uniform cross section under an applied E field.

→ The electric field applied is uniform and its magnitude is given by

$$E = \frac{V}{l}$$

Since the Conductor has a uniform cross section

$$J = \frac{I}{S}$$

$$J = \frac{I}{S} = \sigma E = \frac{\sigma V}{l}$$

Hence $$\boxed{R = \frac{V}{I} = \frac{l}{\sigma S}}$$

(or) $$R = \frac{\rho_c \, l}{S}$$

where $\rho_c = \frac{1}{\sigma}$ is the resistivity of the material.

(P) A wire of diameter 1mm and Conductivity 5×10^7 S/m has 10^{29} free electrons per cubic meter when an electric field of 10m V/m is applied. Determine

(a) The charge density of free electrons

(b) The Current density

(c) The Current in the wire

(d) The drift velocity of the electrons (take the electronic charge as $e = -1.6 \times 10^{-19}$ C)

Sol:

(a) $\rho_v = ne = (10^{29})(-1.6 \times 10^{-19}) = -1.6 \times 10^{10}$ C/m³

(b) $J = \sigma E = (5 \times 10^7)(10 \times 10^{-3}) = 500$ KA/m²

(c) $I = JS = (5 \times 10^5)\left(\frac{\pi d^2}{4}\right) = 0.393$ A.

(d) Since $J = \rho_v u$, $u = \frac{J}{\rho_v} = \frac{5 \times 10^5}{1.6 \times 10^{10}} = 3.125 \times 10^{-5}$ m/s.

Classification of Currents :

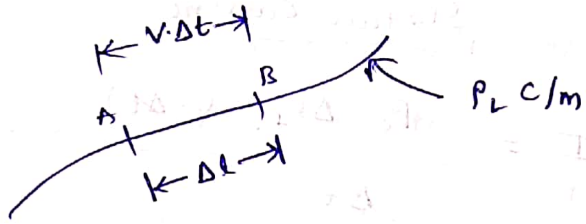
For theoritical Convenience, Currents can be classified in to three types

(a) Line Currents

(b) Surface Currents

(c) Volume Currents.

(a) Line Currents



→ Motion of electric Charges along a line represents "line Current"

→ The amount of mobile charge Contained at any instant with in the elementary Segment is $\rho_L (V \cdot \Delta t)$,

where V = Velocity of Charges

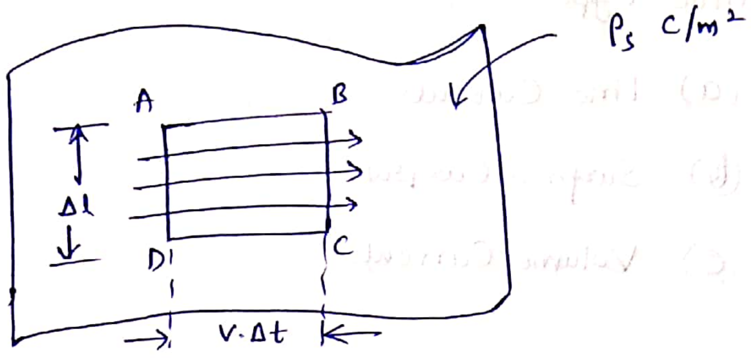
→ All these mobile Charges Coming out of segment in Δt seconds is called Current.

$$I = \frac{\rho_L (V \cdot \Delta t)}{\Delta t}$$

$$\boxed{\bar{I} = \rho_L \bar{V}}$$

where $\rho_L = \frac{I}{V}$ = mobile charge density.

(b) Surface currents:



→ Flow of electric charges over a surface represents Surface currents.

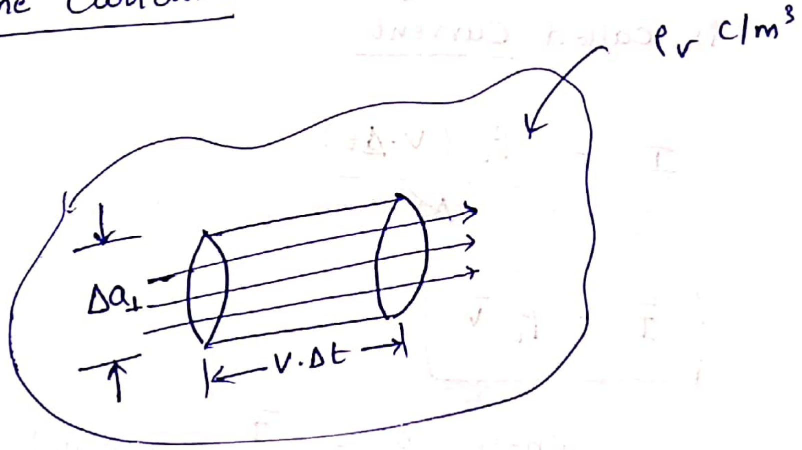
$$\Delta I = \frac{\rho_s \, \Delta l_\perp \cdot (V \cdot \Delta t)}{\Delta t}$$

$$\Delta I = \rho_s V \cdot \Delta l_\perp$$

$$\frac{\Delta I}{\Delta l_\perp} = \rho_s V = K$$

$$\boxed{\bar{K} = \rho_s \cdot \bar{V}}$$ A/m ⇒ where $\bar{K}$ = Surface Current density (A/m)

(c) Volume Currents:



→ Flow of electric charges over a Volume represents Volume currents.

$$\Delta I = \frac{\rho_v \cdot \Delta a_t (v \cdot \Delta t)}{\Delta t}$$

$$\boxed{\frac{\Delta I}{\Delta a_t} = \rho_v \cdot v = J}$$

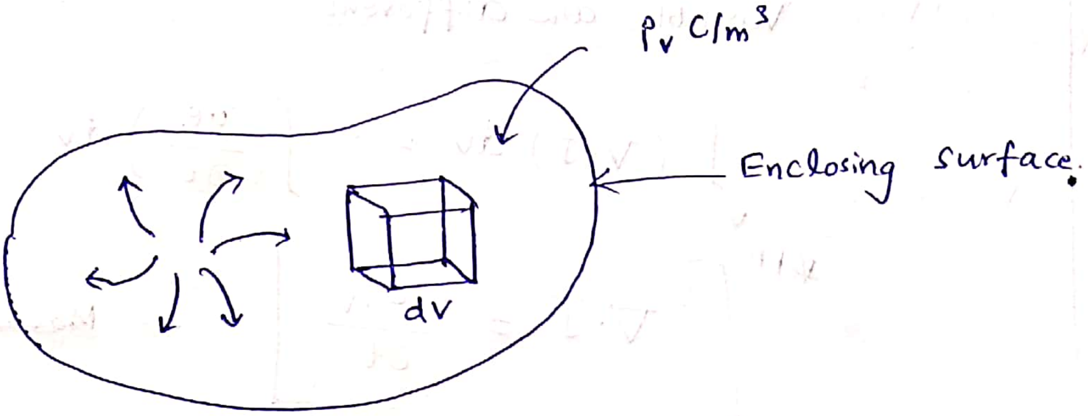
$$\boxed{\bar{J} = \rho_v \cdot \vec{v}} \quad A/m^2$$

where $\bar{J}$ = Volume Current density, A/m^2.

Continuity Equation:

→ The Continuity equation of the Current is based on the principle of Conservation of Charge.

→ This principle states that
" Charges can neither be created nor be destroyed but the charges can transfer from one place to another place".



Let us consider a region carrying Volume Currents. For Convenience let the charges flow outward.

The net outward Current through the enclosing surface Can be obtained as

$$\boxed{I = \oint\!\!\oint_S \bar{J} \cdot d\bar{a}} \rightarrow ① \quad \text{[From volume Currents]}$$

And also, the rate of reduction of electric charges with in the encloser $= -\frac{d}{dt}\int_V \rho_v \cdot dv \rightarrow (2)$

According to the Law of Conservation of Charges, the above two equations are equal

$$\oiint_S \bar{J}\cdot d\bar{a} = -\frac{d}{dt}\int_V \rho_v \cdot dv$$

According to fundamental Theorem of Divergence

$$\int_V (\nabla\cdot\bar{J})\, dv = -\frac{\partial}{\partial t}\int_V \rho_v \cdot dv$$

[Ord D.F = partial Diff when only one variable]

Integration is done w.r.t volume and differentiation is done w.r.t time.

Therefore, $\frac{\partial}{\partial t}$ can be taken inside the integral, since the variables are different.

$$\int_V (\nabla\cdot\bar{J})\, dv = -\int\left(\frac{\partial \rho_v}{\partial t}\right) dv.$$

$$\boxed{\nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t}}$$

← ~~Maxwell's~~

↑ Continuity of Current equation (or) Continuity equation.

→ For steady (DC) currents, $\frac{\partial \rho_v}{\partial t} = 0$, and hence

$\boxed{\nabla\cdot\bar{J} = 0}$ ⇒ Showing the total charge leaving a volume is equal to the total charge entering it.

so $\bar{J}$ is a Solenoid Vector

Effect of introducing charge at some interior point of a given material (Conductor or dielectric) :

we know
point form of Ohm's law

$$J = \sigma E$$

we know Gauss's law

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon}$$

Continuity equation

$$\nabla \cdot J = \frac{-\partial \rho_v}{\partial t}$$

$$\nabla \cdot \sigma E = \frac{\sigma \rho_v}{\varepsilon} = \frac{-\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0$$

This is a homogeneous linear ordinary differential equation.

By seperating variables
we get

$$\frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\varepsilon} \partial t$$

and integrating on both sides

$$\ln \rho_v = -\frac{\sigma t}{\varepsilon} + \ln \rho_{vo}$$

where $\ln \rho_{vo}$ is a constant of integration. Thus

$$\boxed{\rho_v = \rho_{vo} \cdot e^{-t/T_r}}$$

where

$$\boxed{T_r = \frac{\varepsilon}{\sigma}}$$

and T_r is the time constant in seconds.

→ ρ_{vo} is the initial charge density (i.e., ρ_v at $t=0$).

→ The equation shows that the introduction of charge at some interior point of the material results in a decay of volume charge density ρ_v.

→ Associated with the decay is charge movement from the interior point at which it was introduced to the surface of the material.

→ The time constant T_r is known as the "relaxation time" or "rearrangement time".

"Relaxation time is the time it takes a charge placed in the interior of a material to drop to e^{-1} (= 36.8%) of its initial value".

→ Relaxation time is ~~good~~ short for good conductors and long for good dielectrics

Ex: For copper, $\sigma = 5.8 \times 10^7$ S/m, $\varepsilon_r = 1$ and

$$T_r = \frac{\varepsilon_0 \varepsilon_r}{\sigma} = \frac{1 \times 10^{-9}}{36\pi} \times \frac{1}{5.8 \times 10^7}$$

$$= 1.53 \times 10^{-19} \text{ s}$$

showing a rapid decay of charge placed inside a copper.

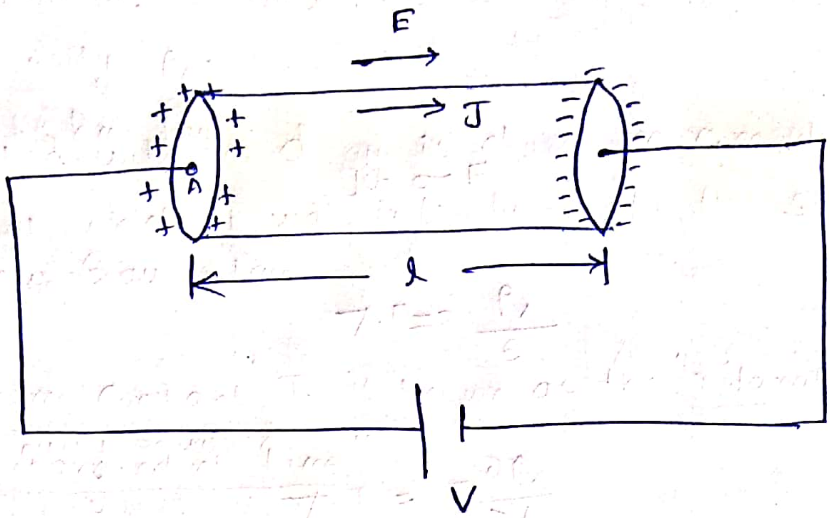
→ Thus for good conductors, the relaxation time is so short that most of the charge will vanish from any interior point and appear at the surface almost instantaneously.

→ For fused quartz, $\sigma = 10^{-17}$ S/m, $\varepsilon_r = 5.0$

$$T_r = 5 \times \frac{10^{-9}}{36\pi} \times \frac{1}{10^{-17}} = 51.2 \text{ days.}$$

showing a very large relaxation time. Thus for good dielectrics, one may consider the introduced charge to remain wherever placed for times up to days.

point form of Ohm's Law :



Current flowing through a Conductor is directly proportional to the potential difference across it, provided temperature is Kept Constant.

$$I \propto V$$

The proportionality Constant is Conductance

$$I = GV$$

$$= \frac{V}{R}$$

$$= \frac{V}{\frac{\rho l}{A}}$$

$$I = \frac{VA}{\rho l}$$

$$\frac{I}{A} = \frac{V}{l} \times \frac{1}{\rho}$$

$$\boxed{\bar{J} = \sigma \bar{E}} \qquad \left[\frac{1}{\rho} = \sigma\right]$$

The above equation is called point form or field form of Ohm's law.

Energy density in Electrostatic fields :

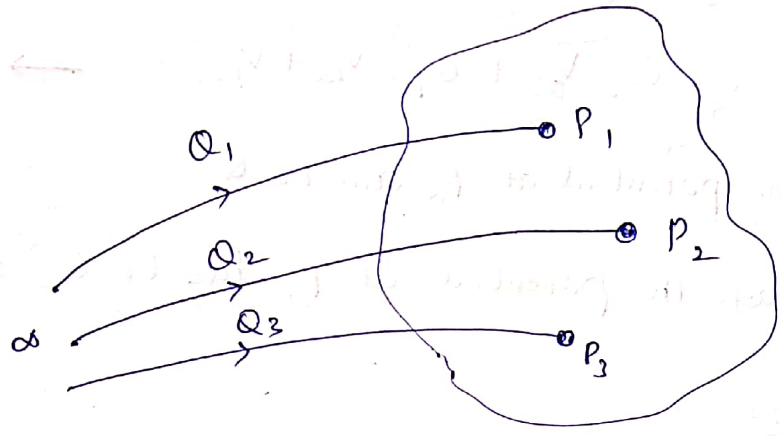


Fig: Assembling of Charges.

→ To determine the energy present in an assembly of Charges, we must first determine the amount of work necessary to assembly them.

→ Suppose we wish to position three point charges Q_1, Q_2 & Q_3 in an initialy empty space shown shaded in figure.

→ No work is required to transfer Q_1 from ∞ to P_1 because the space is initialy charge free and there is no electric field.

→ The workdone in tranferring from infinity to P_2 is equal to the product of Q_2 and the potential V_{21} at P_2 due to Q_1

→ Similarly, the workdone in positioning Q_3 at P_3 is equal to $Q_3(V_{32} + V_{31})$, where V_{32} and V_{31} are the potentials at P_3 due to Q_2 and Q_1, respectively

Hence, the total workdone in positioning the three charges is

$$W_E = W_1 + W_2 + W_3$$

$$= 0 + Q_2 V_{21} + Q_3 (V_{31} + V_{32}) \longrightarrow (1)$$

If the charges are positioned in reverse order

$$W_E = W_3 + W_2 + W_1$$

$$= 0 + Q_2 V_{23} + Q_1 (V_{12} + V_{13}) \longrightarrow (2)$$

Where V_{23} is the potential at P_2 due to Q_3

V_{12} & V_{13} are the potentials at P_1 due to Q_2 & Q_3.

Adding (1) & (2)

$$2W_E = Q_1(V_{12} + V_{13}) + Q_2(V_{21} + V_{23}) + Q_3(V_{31} + V_{32})$$

$$= Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

$$\boxed{W_E = \frac{1}{2}(Q_1 V_1 + Q_2 V_2 + Q_3 V_3)} \longrightarrow (3)$$

where V_1, V_2 & V_3 are total potentials at P_1, P_2 & P_3 respectively.

→ In general, if there are 'n' point charges, Eq (3) becomes

$$\boxed{W_E = \frac{1}{2}\sum_{K=1}^{n} Q_K V_K} \text{ (in Joules).} \longrightarrow (4)$$

→ Instead of point charges, the region has a Continuous Charge distribution, the Summation in Eq (4) becomes integration,

$$W_E = \frac{1}{2}\int_L \rho_L \cdot V \cdot dl \text{ (Line charge)} \longrightarrow (5)$$

$$W_E = \frac{1}{2}\int_S \rho_s \cdot V \cdot ds \text{ (Surface charge)} \longrightarrow (6)$$

$$W_E = \frac{1}{2}\int_V \rho_v \cdot V \cdot dv \text{ (Volume charge)} \longrightarrow (7)$$

Since, $\nabla \cdot D = \rho_v$ in Eq (7)

$$W_E = \frac{1}{2}\int_V (\nabla \cdot D) \cdot V \cdot dv \rightarrow (8)$$

But for any vector $\bar{A}$ and scalar V, the identity

$$\nabla \cdot VA = A \cdot \nabla V + V \cdot (\nabla \cdot A)$$

(or)

$$(\nabla \cdot A)V = \nabla \cdot VA - A \cdot \nabla V$$

Applying this identity to Eq (8)

$$W_E = \frac{1}{2}\int_V (\nabla \cdot VD)\, dv - \frac{1}{2}\int_V (D \cdot \nabla V)\, dv \rightarrow (9)$$

By Applying divergence theorem to the first term on the right-hand side of this equation, we have

$$W_E = \frac{1}{2}\oint_S (VD) \cdot ds - \frac{1}{2}\int (D \cdot \nabla V)\, dv \rightarrow (10)$$

As we know V varies as $\frac{1}{r}$ and D as $\frac{1}{r^2}$ for point charges

V varies as $\frac{1}{r^2}$ and D as $\frac{1}{r^3}$ for dipoles.

VD must vary $\frac{1}{r^3}$ while ds varies as r^2

Consequently, the first integral must tend to zero as the surface S becomes large.

Hence Eq (10) becomes

$$W_E = -\frac{1}{2}\int_V (D \cdot \nabla V)\, dv = \frac{1}{2}\int_V (D \cdot E) \cdot dv.$$

and since $E = -\nabla V$ and $D = \varepsilon_0 E$,

the electrostatic energy is

$$\boxed{W_E = \frac{1}{2}\int_V D\cdot E\cdot dv = \frac{1}{2}\int_V \varepsilon_0 E^2\cdot dv} \rightarrow (11)$$

From this, we can define electrostatic energy density, w_E (in J/m^3) as

$$\boxed{w_E = \frac{dW_E}{dv} = \frac{1}{2}\cdot D\cdot E = \frac{1}{2}\varepsilon E^2 = \frac{D^2}{2\varepsilon_0}} \rightarrow (12)$$

Eq

$$\boxed{W_E = \int_V w_E\cdot dv} \rightarrow (13)$$

P) The point charges $-1nC$, $4nC$ and $3nC$ are located at $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$, respectively. Find the energy in the system.

Sol: Method (1):

$$W = W_1 + W_2 + W_3$$

$$= 0 + Q_2 V_{21} + Q_3(V_{31} + V_{32})$$

$$= Q_2\cdot\frac{Q_1}{4\pi\varepsilon_0|(0,0,1)-(0,0,0)|}$$

$$+ \frac{Q_3}{4\pi\varepsilon_0}\left[\frac{Q_1}{|(1,0,0)-(0,0,0)|} + \frac{Q_2}{|(1,0,0)-(0,0,1)|}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[Q_1Q_2 + Q_1Q_3 + \frac{Q_2Q_3}{\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\times\frac{10^{-9}}{36\pi}}\left[-4-3+\frac{12}{\sqrt{2}}\right]\cdot 10^{-18}$$

$$= 13.37\ nJ$$

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BOUNDARY CONDITIONS

If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface seperating the media are called boundary conditions.

→ These conditions are helpful in determining the field on one side of the boundary if the field on the other side is known.

→ We consider the boundary conditions at the interface seperating

- Dielectric (ε_{r_1}) and dielectric (ε_{r_2})
- Conductor and dielectric
- Conductor and free space.

→ To determine the boundary conditions, we need to use Maxwell's equations.

$$\oint E \cdot dl = 0 \longrightarrow (1)$$

and

$$\oint D \cdot ds = Q_{enc} \longrightarrow (2)$$

where Q_{enc} is the free charge enclosed by the surface S.

Also we need to decompose the electric field Intensity $\bar{E}$ in to two orthogonal components:

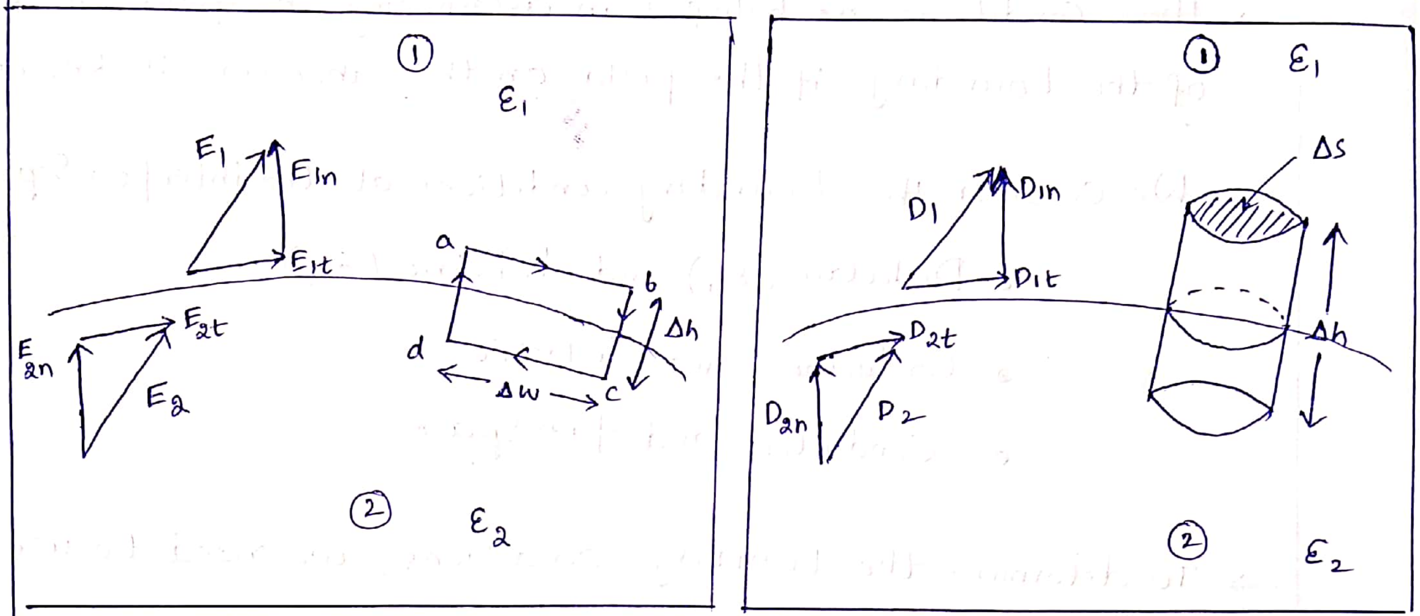
$$\bar{E} = \bar{E}_t + \bar{E}_n$$

Where $\bar{E}_t$ and $\bar{E}_n$ are, respectively, the tangential and normal components of $\bar{E}$ to the interface of interest.

$$\bar{D} = \bar{D}_t + \bar{D}_n .$$

(1) Dielectric – Dielectric Boundary Conditions :

Consider the $\bar{E}$ field existing in a region that consists of two different dielectrics characterized by $\varepsilon_1 = \varepsilon_0 \varepsilon_{r_1}$ and $\varepsilon_2 = \varepsilon_0 \varepsilon_{r_2}$ as shown in figure (1).



(a) (b)

Fig(1) : Dielectric – dielectric boundary
(a) determining $E_{1t} = E_{2t}$
(b) determining $D_{1n} = D_{2n}$

→ The fields E_1 and E_2 in media 1 and 2, respectively, can be decomposed as

$$E_1 = E_{1t} + E_{1n} \longrightarrow (3)$$

$$E_2 = E_{2t} + E_{2n} \longrightarrow (4)$$

We apply Eq(1) to the closed path abcda of fig(1)(a), assuming that the path is very small w.r.t the spatial variation of E. We obtain

$$0 = E_{1t}\,\Delta w - \cancel{E_{1n}\frac{\Delta h}{2}} - \cancel{E_{2n}\frac{\Delta h}{2}} - E_{2t}\,\Delta w + \cancel{E_{2n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}}$$

$$\Rightarrow \quad 0 = (E_{1t} - E_{2t})\,\Delta w$$

$$\boxed{E_{1t} = E_{2t}} \rightarrow (5)$$

→ Thus the tangential components of $\bar{E}$ are the Same on the two sides of the boundary.

In otherwords, E_t undergoes no change on boundary and it is said to be continuous across the boundary.

Since $D = \varepsilon E = D_t + D_n$

Eq (5) can be written as

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}.$$

$$\boxed{\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}} \longrightarrow (6)$$

→ i.e., D_t undergoes some change across the interface. Hence D_t is said to be disontinuous across the Surface.

Similarly, appply Eq (2) to the pill box (cylindrical Gaussian surface) of fig (1)(b).

The Contribution due to the sides vanishes. Allowing $\Delta h \to 0$ gives

$$\Delta Q = \rho_s \cdot \Delta s = D_{1n} \Delta s - D_{2n} \Delta s$$

$$\boxed{D_{1n} - D_{2n} = \rho_s} \longrightarrow (7)$$

where ρ_s is the free charge density placed deliberately at the boundary. ~~Et should~~

If no free charges exist at the interface (i.e., charges are not deliberately placed there), $\rho_s = 0$ and Eq (7) becomes

$$\boxed{D_{1n} = D_{2n}} \rightarrow (8)$$

Thus, the normal components of $\bar{D}$ is continuous across the interface; that is, D_n undergoes no change at the boundary.

Since $D = \varepsilon E$, Eq (8) can be written as

$$\boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}} \rightarrow (9)$$

showing that the normal component of $\bar{E}$ is discontinuous at the boundary.

→ Equations (5) & (8) are collectively referred to as boundary conditions.

→ As mentioned earlier, the boundary conditions are usually applied in finding the electric field on one side of the boundary given the field on other side.

~~We can~~

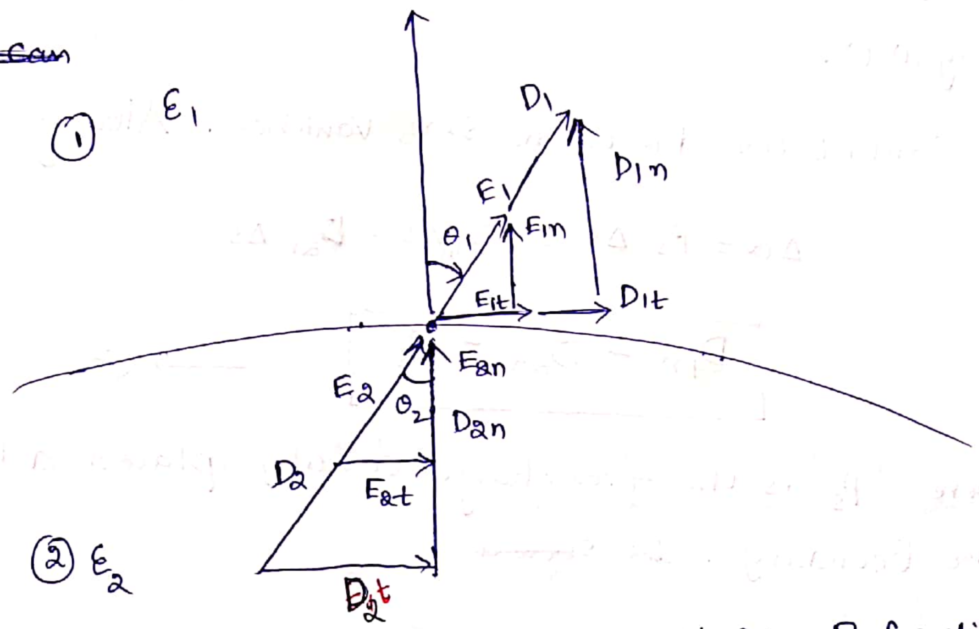


Fig (2): Refraction of $\bar{D}$ or $\bar{E}$ at a dielectric-dielectric boundary.

→ We use the boundary conditions to determine the "refraction" of the electric field across the interface.

Consider D_1 or E_1 and D_2 or E_2 making angles θ_1 and θ_2 with the normal to the interface as illustrated in fig (2).

Using Eq (5), we have

$$E_1 \sin\theta_1 = E_{1t} = E_{2t} = E_2 \sin\theta_2 .$$

$$\boxed{E_1 \sin\theta_1 = E_2 \sin\theta_2} \rightarrow (10)$$

Similarly, by applying eq (8), we get

$$\varepsilon_1 E_1 \cos\theta_1 = D_{1n} = D_{2n} = \varepsilon_2 E_2 \cos\theta_2$$

$$\Rightarrow \boxed{\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2} \rightarrow (11)$$

$$\text{Eq (10) / Eq (11)} \Rightarrow \frac{\cancel{E_1} \sin\theta_1}{\varepsilon_1 \cancel{E_1} \cos\theta_1} = \frac{\cancel{E_2} \sin\theta_2}{\varepsilon_2 \cancel{E_2} \cos\theta_2}$$

$$\boxed{\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}} \rightarrow (12)$$

Since $\varepsilon_1 = \varepsilon_0 \varepsilon_{r_1}$ and $\varepsilon_2 = \varepsilon_0 \varepsilon_{r_2}$

$$\boxed{\frac{\tan\theta_1}{\varepsilon_0 \varepsilon_{r_1}} = \frac{\tan\theta_2}{\varepsilon_0 \varepsilon_{r_2}}} \rightarrow (13)$$

→ This is the law of refraction of the electric field at the boundary free of charge (since $\rho_s = 0$ is assumed at the interface).

→ Thus, in general, an interface b/w two dielectrics produces bending of the flux lines as a result of unequal polarization charges that accumulate on the opposite sides of interface.

(2) Conductor - Dielectric Boundary Conditions

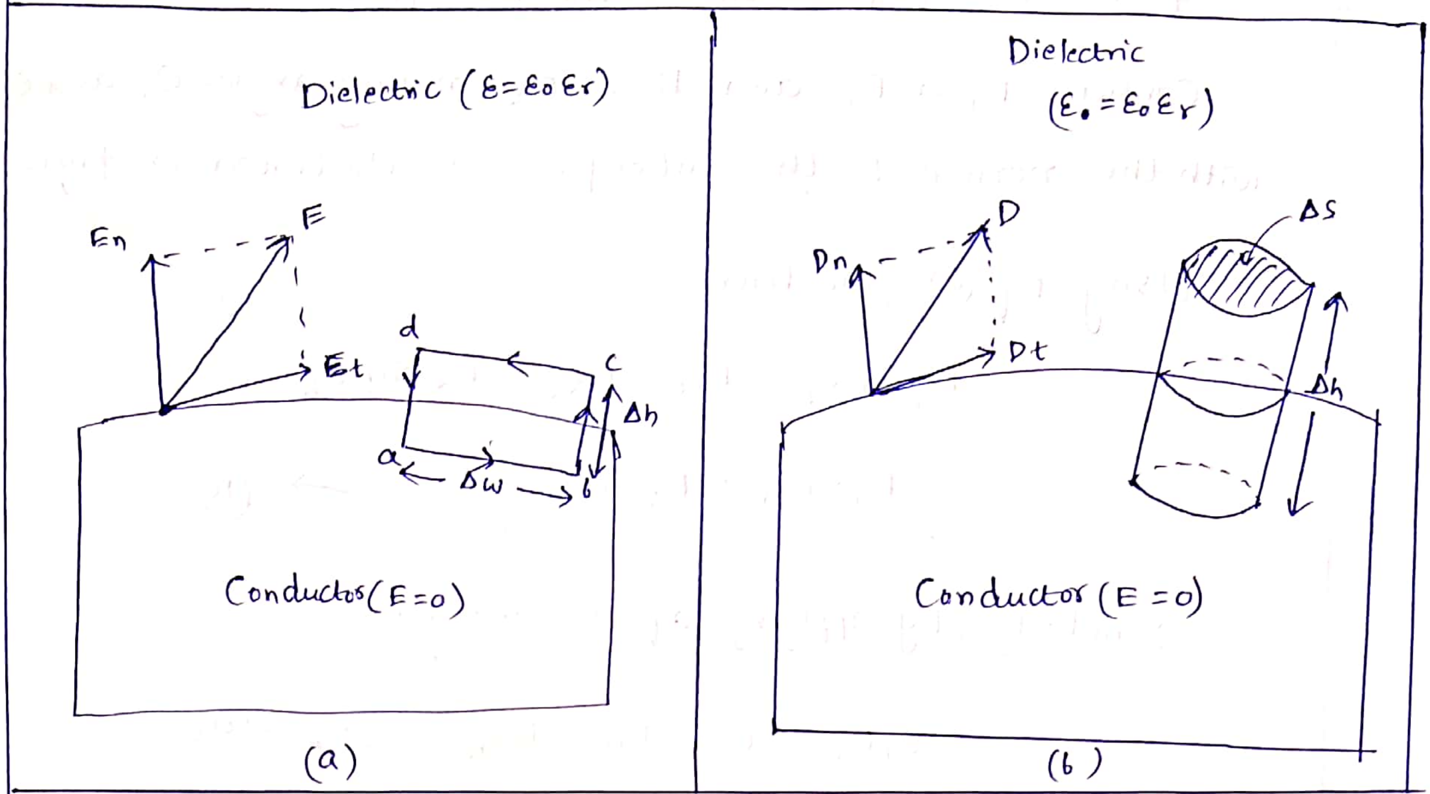


Fig (3): Conductor - dielectric boundary.

→ The conductor is assumed to be perfect (i.e., $\sigma \to \infty$)

To determine the boundary conditions for a conductor-dielectric interface, we follow the same procedure used for the dielectric-dielectric interface except we incorporate the fact that $E=0$ inside the conductor.

Apply Eq (1) to the closed path abcda of fig(3)(a) gives

$$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2}$$

⇒ As $\Delta h \to 0$,

$$\boxed{E_t = 0} \to (14)$$

Similarly, Apply Eq (2) to the cylindrical pill box of Fig (3)(b) and letting $\Delta h \to 0$,

we get $\Delta Q = D_n \cdot \Delta s - 0 \cdot \Delta s$

$$\Rightarrow \quad D_n = \frac{\Delta Q}{\Delta s} = \rho_s$$

$$\boxed{D_n = \rho_s} \quad \rightarrow (15).$$

NOTE

Thus under static Conditions, the following Conclusions Can be made about a perfect Conductor:

(1) No electric field may exist within a conductor

$$\boxed{\rho_v = 0, \quad E = 0} \quad \rightarrow (16)$$

(2) Since $E = -\nabla V = 0$, there can be no potential difference b/w any two points in the Conductor i.e., Conductor is an equipotential body.

(3) An electric field $\bar{E}$ must be external to the Conductor and must be normal to its surface

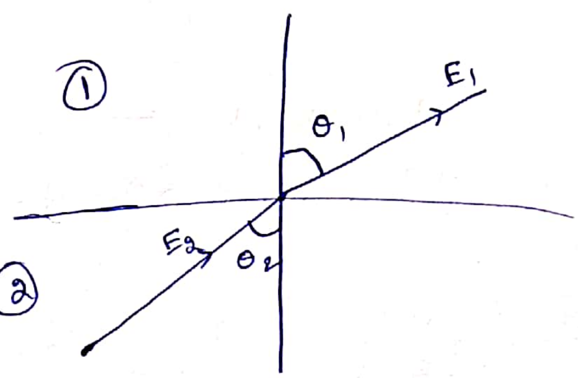
i.e.,

$$\boxed{\begin{aligned} D_t &= \varepsilon_0 \varepsilon_r E_t = 0 \\ D_n &= \varepsilon_0 \varepsilon_r E_n = \rho_s \end{aligned}} \quad \rightarrow (17)$$

(3)

(P) Calculate the emerging angle by which the Vector E changes its direction as it passes from a medium with $\varepsilon_r = 100$ in to air making an angle of $45°$ with the interface as it enters.

Sol: $\varepsilon_{r_2} = 100, \ \varepsilon_{r_1} = 1, \ \theta_2 = 45°$



$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}$$

$$\Rightarrow \frac{\tan\theta_1}{\tan 45°} = \frac{1}{100}$$

$$\Rightarrow \boxed{\theta_1 = 0.57°}$$

(3) Conductor - Free space Boundary Conditions :

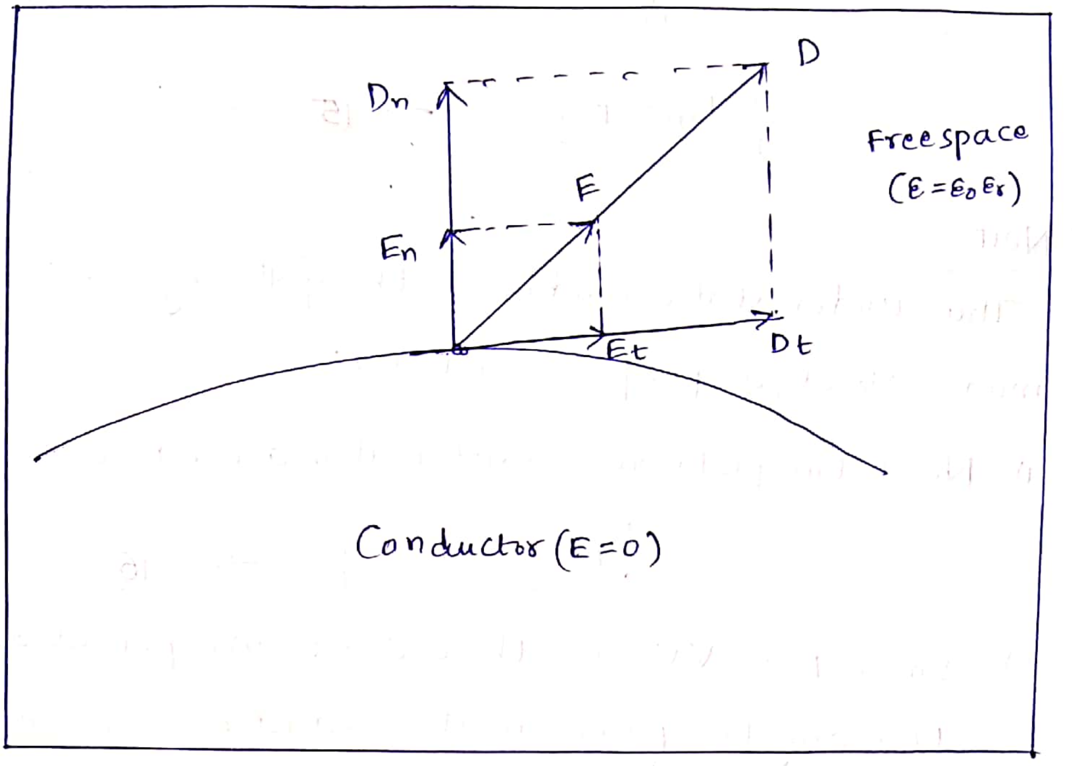


Fig (4): Conductor - Free space boundary

→ The boundary conditions at the interface b/w a conductor and free space can be obtained from Eq (17) by replacing ε_r by 1.

Thus the boundary conditions are

$$\boxed{\begin{aligned} D_t &= \varepsilon_0 E_t = 0 \\ D_n &= \varepsilon_0 E_n = \rho_s \end{aligned}} \longrightarrow (18)$$

Capacitance - Capacitors :

→ A capacitor is an electrical device composed of two conductors which are seperated through a delectric medium and which can store equal and opposite charges ($\pm Q$), independently of whether other conductors in the system are charged or not.

→ The capacitance between two conductors is defined by the simple relation

$$\boxed{C = \frac{Q}{V}}$$

C

where C = capacitance in farads

Q = charge in Coulomb on each conductor

V = potential difference b/w the conductors due to the equal and opposite charges on them of magnitude Q.

(1) Parallel - plate Capacitor :

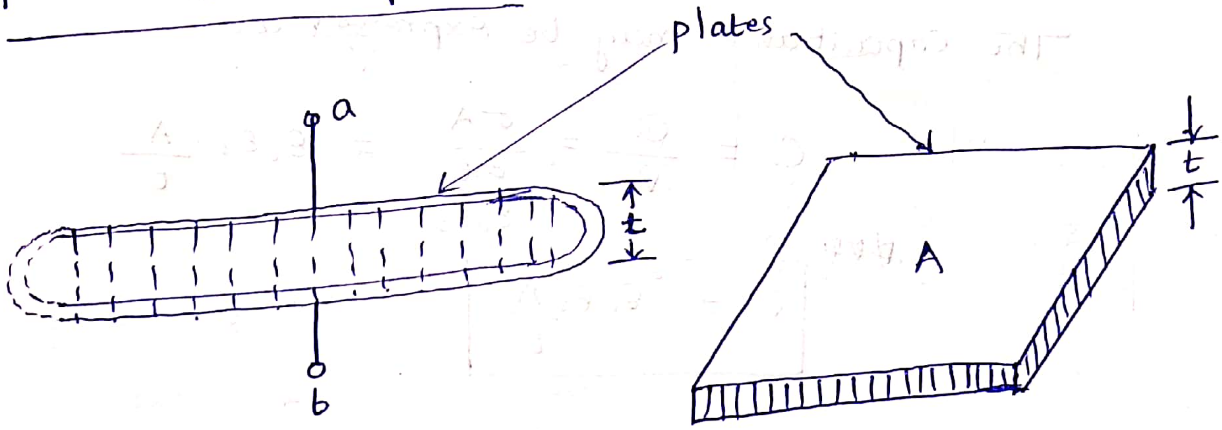


Fig: parallel-plate capacitor

→ The figure shows the typical parallel-plate capacitor which consists of a pair of flat parallel plates with surface area A seperated by a distance "t" and through a dielectric of permitivity $\varepsilon = \varepsilon_0 \varepsilon_r$.

→ The capacitor may be charged by connecting the terminals a and b to a source of p.d.

→ Let us assume that there is a uniform charge density over the plate surface, σ or ρ_s C/m^2 and also across the dielectric.

$$D = \sigma \text{ or } \rho_s = \frac{Q}{A} \longrightarrow (1)$$

So, that the field intensity is

$$E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \longrightarrow (2)$$

→ We know the relation

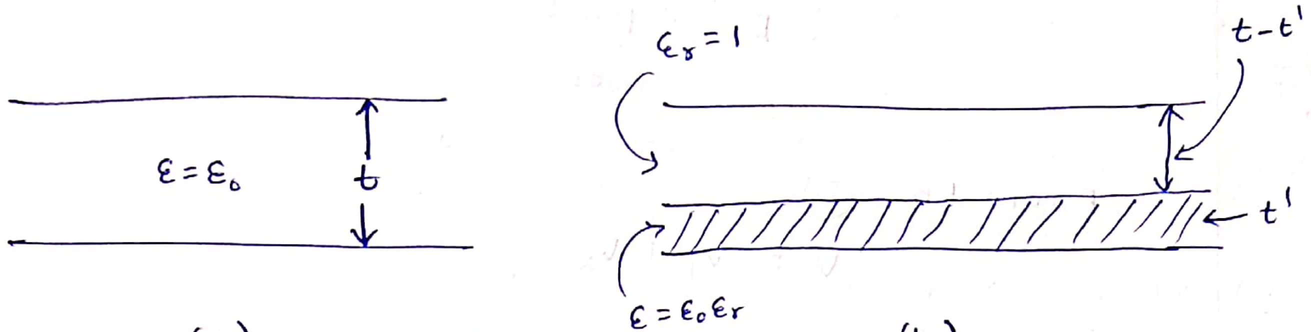
$$V = Et = \frac{\sigma t}{\varepsilon_0 \varepsilon_r} \longrightarrow (3)$$

The capacitance may be expressed as

$$C = \frac{Q}{V} = \frac{\cancel{\sigma} A}{\frac{\cancel{\sigma} t}{\varepsilon_0 \varepsilon_r}} = \varepsilon_0 \varepsilon_r \cdot \frac{A}{t}$$

$$\boxed{C = \varepsilon_0 \varepsilon_r \frac{A}{t}}$$

(2) Capacitance of a parallel plate Capacitor having two dielectric media :



(a) (b)

→ A parallel plate Capacitor has a plate Seperation 't'. The capacitance with air only between the plates is C. shown in fig (a).

→ When a slab of thickness t' and relative permittivity ε_r is placed on one of the plates, the Capacitance is C'.

→ With air alone as dielectric,

$$C = \varepsilon_0 \frac{A}{t} \quad (\text{as } \varepsilon_r = 1) \longrightarrow (1)$$

→ Now, with the introduction of a slab of thickness t', the thickness of air-film is reduced to $(t-t')$.

→ Let E_1 and E_2 be the field intensities in the air film and slab respectively. V_1 and V_2 the potential differences across them.

→ If V is the P.D. across the Capacitor,

$$V = V_1 + V_2 \longrightarrow (2)$$

$$\left.\begin{aligned} V_1 &= E_1 (t-t') \\ V_2 &= E_2 t' \end{aligned}\right\} \longrightarrow (3)$$

As we Know: $D = \frac{Q}{A}$

$$E_1 = \frac{D}{\epsilon_0}$$

$$E_2 = \frac{D}{\epsilon_0 \epsilon_r}$$

From Eq ②

$$V = V_1 + V_2$$

$$= E_1 (t-t') + E_2 t'$$

$$= \frac{D}{\epsilon_0}(t-t') + \frac{D}{\epsilon_0 \epsilon_r} t'$$

$$= \frac{Q}{A\epsilon_0}(t-t') + \frac{Q}{A\epsilon_0\epsilon_r} t'$$

$$= \frac{Q}{A\epsilon_0}\left[\frac{1}{\epsilon_r} t' + (t-t')\right]$$

The new Value of Capacitance

$$C' = \frac{Q}{V}$$

$$= \frac{A\epsilon_0}{\frac{1}{\epsilon_r} t' + (t-t')}$$

$$\boxed{C' = \frac{A\epsilon_0\epsilon_r}{t' + \epsilon_r(t-t')}} \longrightarrow ④$$

From Eqns ① & ④

$$\frac{C'}{C} = \frac{\frac{A\epsilon_0\epsilon_r}{t'+\epsilon_r(t-t')}}{\epsilon_0 \cdot \frac{A}{t}} = \frac{\cancel{A\epsilon_0}\epsilon_r}{t'+\epsilon_r(t-t')} \times \frac{t}{\cancel{\epsilon_0 A}}$$

$$\boxed{\frac{C'}{C} = \frac{\varepsilon_r t}{t' + \varepsilon_r (t - t')}}$$

(3). Capacitance of an isolated sphere :

→ In case of isolated Conductor, the other Conductor forming part of the Capacitor is a spherical shell of infinite radius.

→ Let its radius be r_1. The potential of the isolated sphere is the workdone per unit charge in carrying a positive test charge from infinity to the sphere.

→ The absolute potential is

$$V = \frac{Q}{4\pi\varepsilon_0 r_1}$$

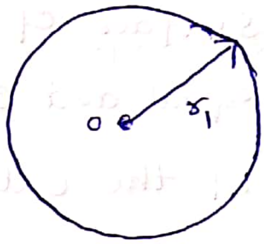


Fig: Isolate sphere.

→ For free space, the Capacitance is given by

$$C = \frac{Q}{V}$$

$$= \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0 r_1}} = 4\pi\varepsilon_0 r_1$$

$$\boxed{C = 4\pi\varepsilon_0 r_1}$$

(4) Capacitance between two Concentric Spherical Shells :-

→ A spherical capacitor is composed of two concentric, spherical conducting shells seperated through a dielectric medium, say free space in this case.

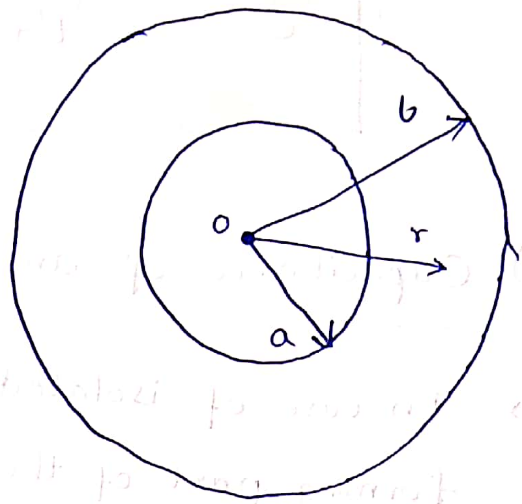


Fig: Capacitance of Spherical Capacitor

→ Let a and b be the radii of inner and outer shells respectively.

→ If a charge Q is distributed uniformly over the Outer surface of the inner shell of radius a, there will be equal and opposite charge induced on the inner surface of the outer shell (radius b).

→ The field at any point between the shells is given by

$$E_r = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (a \le r \le b)$$

→ The potential difference b/w the shells

$$V_{ba} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{b-a}{ab}\right]$$

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The Capacitance of the spherical - shell capacitor

$$C = \frac{Q}{V_{ba}}$$

$$\boxed{C = 4\pi\varepsilon_0 \left[\frac{ab}{b-a}\right]}$$

The Capacitance is thus proportional to the product of the shell radii and inversely proportional to their difference.

(5) Capacitance between Co-axial cylinders :
(Co-axial Cables)

→ Let the dielectric be interposed between the two-coaxial cylinders of radii a (inner) and b (outer) respectively.

→ The potential b/w two co-axial cylinders is

$$V_{ba} = \frac{\rho_L}{2\pi\varepsilon_0\varepsilon_r} \ln\left(\frac{b}{a}\right)$$

where ρ_L or λ is the charge per metre length.

→ The Capacitance between the cylinders per unit length is

$$C_{ab} = \frac{\rho_L}{\frac{\rho_L}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{b}{a}\right)}$$

$$\boxed{C_{ab} = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}}$$

KNL Chary

→ Incidentally, for a long cable we can obtain the capacitance per kilometre length in micro farads.

→ The above equation gives the capacitance in farads per metre length.

$$C_{ab} = \frac{2\pi \varepsilon_0 \varepsilon_r}{2.303 \log_{10} \frac{b}{a}} \times 10^3 \text{ Farad/Km}$$

$$= \frac{2\pi \times \frac{1\times 10^{-9}}{36\pi} \cdot \varepsilon_r}{2.303 \log_{10} \frac{b}{a}} \times 10^3 \text{ Farad/Km}$$

$$\boxed{C_{ab} = \frac{0.0241\ \varepsilon_r}{\log_{10} \frac{b}{a}}} \ \mu F/Km.$$

(6) Capacitance of Co-axial Cable with two dielectrics

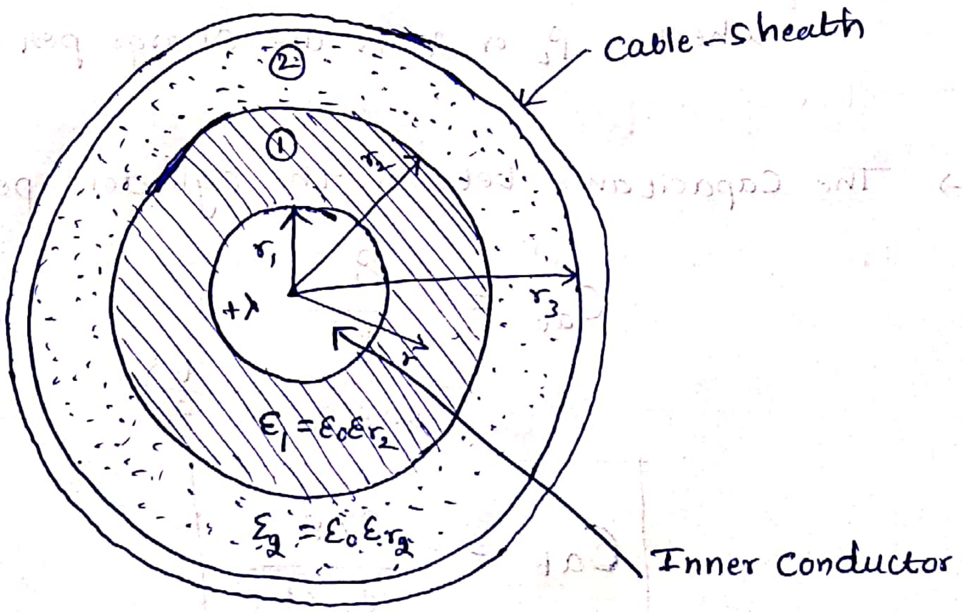


Fig: Capacitance of Co-axial Cable with two dielectrics, with permittivities ε_1 and ε_2.

→ Now Let us Consider a Cable with two dielectrics with permittivities ε_1 and ε_2.

→ If E_1 is the EFI at any radial distance 'r' in the dielectric (1) (inner) and E_2 that in the dielectric (2) (outer).

We have the relations

$$E_1 = \frac{\lambda_0}{2\pi\varepsilon_1 r} \quad (r_1 \le r \le r_2) \longrightarrow (1)$$

$$E_2 = \frac{\lambda}{2\pi\varepsilon_2 r} \quad (r_2 \le r \le r_3) \longrightarrow (2)$$

If then, the p.D's across the dielectrics (1) and (2) are V_1 and V_2 respectively, we have

$$V_1 = -\int_{r_2}^{r_1} E_1 dr \longrightarrow (3)$$

$$V_2 = -\int_{r_3}^{r_2} E_2 dr \longrightarrow (4)$$

Then,
$$V_1 = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r_1}} \ln\frac{r_2}{r_1} \longrightarrow (5)$$

$$V_2 = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r_2}} \ln\frac{r_3}{r_2} \longrightarrow (6)$$

→ The p.D between the inner Conductor and the Cable-Sheath is V, then

$$V = V_1 + V_2$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{r_1}}\ln\frac{r_2}{r_1} + \frac{1}{\varepsilon_{r_2}}\ln\frac{r_3}{r_2}\right] \longrightarrow (7)$$

The capacitance per metre length is

$$C = \frac{\lambda}{V}$$

$$= \frac{0.0241\ \varepsilon_{r_1} \varepsilon_{r_2}}{\varepsilon_{r_2} \log_{10}\frac{r_2}{r_1} + \varepsilon_{r_1} \log_{10}\frac{r_3}{r_2}}$$

For a km length, it will be given by

$$\boxed{C = \frac{0.0241\ \varepsilon_1 \varepsilon_2}{\varepsilon_2 \log_{10}\frac{r_2}{r_1} + \varepsilon_1 \log_{10}\frac{r_3}{r_2}}\ \mu F/km}$$

(7) Capacitance between two parallel wires : (Over head transmission lines)

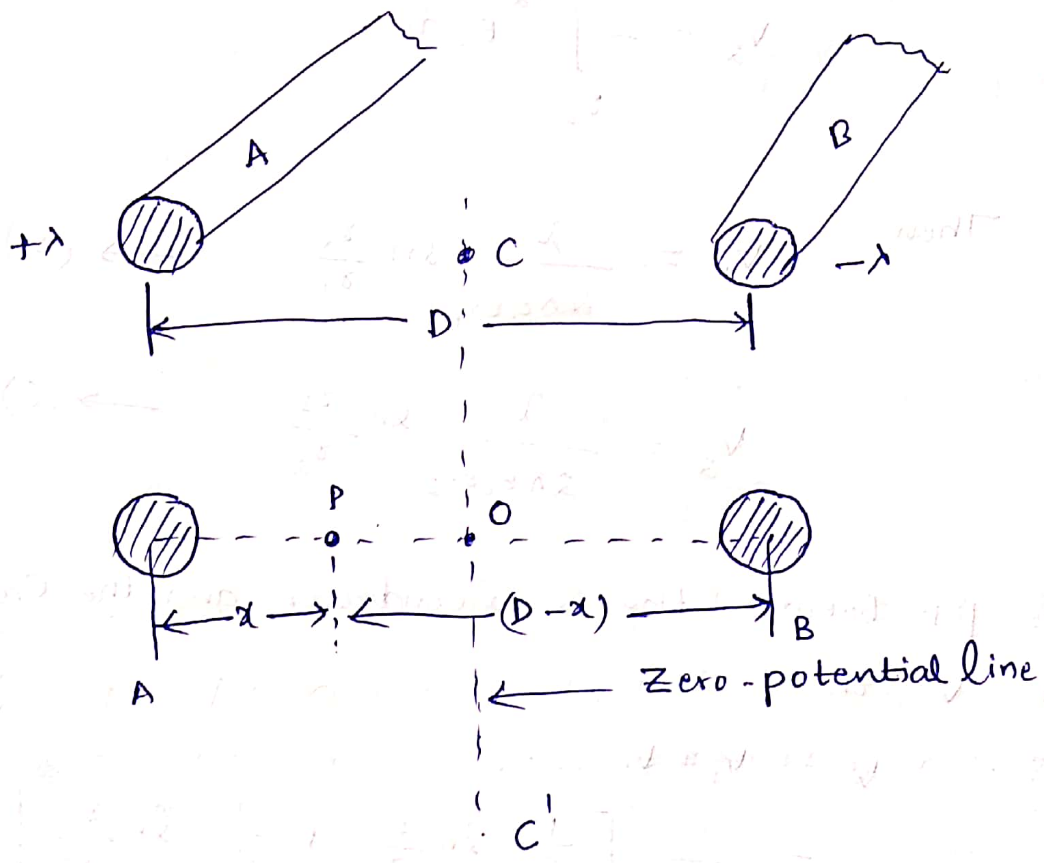


Fig: parallel-wires : capacitance

→ We will assume that $+\lambda$ and $-\lambda$ are the charges in Coulomb per metre length of the wires A and B, spaced 'D' metres apart and radius of each wire is 'r' metre, remembering that $D >> r$.

→ If E_x is the field intensity at any point 'p' on the line distant x from A and $(D-x)$ from B, then

V_{BA} = potential rise from B to A

$$= -\int_{B}^{A} E_x \, dx \longrightarrow (1)$$

→ The limits being the equipotential surfaces of wires B & A,

$x = D - r$

$x = r$ respectively.

→ The field at 'p' is the vector sum of the fields due to A and B.

$$E_x = \frac{\lambda}{2\pi\epsilon_0 x} + \frac{\lambda}{2\pi\epsilon_0 (D-x)} \quad (r \le x \le D-r) \longrightarrow (2)$$

directed from A to B.

→ The potential rises from B to A

$$V = -\frac{\lambda}{2\pi\epsilon_0}\int_{D-r}^{r}\left[\frac{1}{x} + \frac{1}{(D-x)}\right] dx \longrightarrow (3)$$

$$= \frac{-\lambda}{2\pi\epsilon_0}\Big[\ln|x| + \ln|D-x|\Big]_{D-r}^{r}$$

$$V = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln\frac{r}{D-r} + \ln\frac{r}{D-r}\right]$$

$$= \frac{\lambda}{\cancel{2}\pi\varepsilon_0}\cancel{2}\ln\frac{D-r}{r}$$

$$V = \frac{\lambda}{\pi\varepsilon_0}\ln\left[\frac{D-r}{r}\right] \longrightarrow (4)$$

As D is too large in comparision with r,

$$\frac{D-r}{r} \simeq \frac{D}{r}$$

$$\boxed{V = \frac{\lambda}{\pi\varepsilon_0}\ln\left(\frac{D}{r}\right)} \longrightarrow (5)$$

The capacitance between the two wires A and B is,

$$\boxed{C_{AB} = \frac{\lambda}{V} = \frac{\pi\varepsilon_0}{\ln\frac{D}{r}}} \ \mu F/m \longrightarrow (6)$$

$$\boxed{C_{AB} = \frac{0.01205}{\log_{10}\frac{D}{r}} \ \mu F/km} \longrightarrow (7)$$

→ Consider any point on the perpendicular bisector of the line AB. As C is equidistant from equal and opposite charges $\pm\lambda$, the resultant potential at C is zero.

Therefore, the perpendicular bisector is an equipotential line for zero-potential. As the P.D between A and B is V, the potential difference between A and O (midpoint of AB) = P.D between O & B.

$= \frac{V}{2}$, and so the capacitance of each conductor (A or B) per metre length is

$$C_A \text{ or } (C_B) = \frac{\lambda}{V/2} = \frac{2\lambda}{V}$$

$\Rightarrow$ so, C_A or $(C_B) = 2\, C_{AB}$

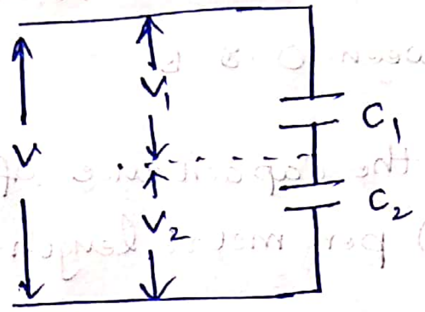
The capacitance per metre length of each conductor is thus ~~that~~ twice that between the conductors.

$$C_A = C_B$$

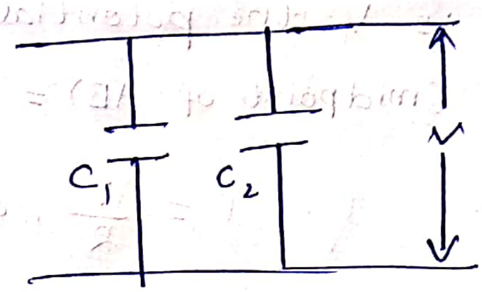
$$= \frac{2\pi \varepsilon_0}{\ln\left(\frac{D}{r}\right)} \quad F/m$$

$$= \frac{0.0241}{\log_{10}\left(\frac{D}{r}\right)} \quad \mu F/km$$

Capacitors in series and in parallel :



(a) Series Connection of C_1 & C_2



(b) parallel Connection of C_1 & C_2.

→ If two capacitors, which initially possess no charge, are joined in series and a potential difference V is applied to the Combination, they are Charged.

$$V_1 = \frac{Q}{C_1} \quad \text{and} \quad V_2 = \frac{Q}{C_2}$$

Where Q is the charge which is the same for both the Capacitors.

Hence $V = V_1 + V_2$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2}\right]$$

$$\Rightarrow \frac{\cancel{Q}}{C} = \cancel{Q}\left[\frac{1}{C_1} + \frac{1}{C_2}\right]$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

For 'n' no. of Capacitors are Connected in parallel

$$\boxed{\frac{1}{C} = \sum_{K=1}^{n} \frac{1}{C_K}}$$

→ In parallel case, the same Voltage V appears across each of the Capacitors, and the total charge is the sum of the charges on the individual Capacitors Q_1 & Q_2.

Thus $Q = Q_1 + Q_2$

$$Q = C_1 V + C_2 V$$

$$\Rightarrow \quad CV = C_1 V + C_2 V$$

$$\Rightarrow \quad \boxed{C = C_1 + C_2}$$

If there are n capacitors,

$$\boxed{C = C_1 + C_2 + \dots + C_n}$$

Energy in a Capacitor:

~~The~~ If a Capacitor is connected to a source of potential, the capacitor acquires charge.

→ potential is

$$V = \frac{dw}{dq}$$

$$dw = V \cdot dq$$

$$dw = \frac{q}{C} \cdot dq.$$

If a Capacitor is initialy uncharged, and the process of charging continued until a charge Q is reached, the total workdone is

$$W = \frac{1}{C} \int_0^Q q \cdot dq$$

$$W = \frac{Q^2}{2C}$$

Where $Q = CV$

$$\boxed{W = \frac{Q^2}{2C} \text{ or } \frac{1}{2}CV^2 \text{ or } \frac{1}{2}QV}$$

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CAPACITANCE

PROBLEMS

Capacitance (problems)

(1) Obtain the capacitance of an isolated conducting sphere of radius 1 cm.

Sol) Given data

Radius, $r = 1\,cm$

$= 0.01\,m.$

For an isolated sphere,

Absolute potential, $V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r}.$

Capacitance, $C = \frac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r r$

$= 4\pi \times 8.854 \times 10^{-12} \times 1 \times 0.01$

$= 1.11\ pF.$

(2) Find the capacitance of parallel plate capacitor when $A = 1\,m^2$ distance b/w the plates 1mm, Voltage gradient is $10^5\ V/m$ and charge on the plate is $2\,\mu C/m^2$.

Sol) Given data

Area covered by the plates,

$A = 1\,m^2$

Distance between the plates,

$d = 1\,mm = 1 \times 10^{-3}\,m$

Voltage gradient, $E = 10^5$ V/m

charge density on plates, $\rho = 2\ \mu C/m^2$

$= 2 \times 10^{-6}\ C/m^2$

Capacitance, $C = ?$

From the definition of capacitance, we have,

$$\text{Capacitance, } (C) = \frac{\text{Charge } (Q)}{\text{Voltage } (V)}$$

i.e., $\boxed{C = \frac{Q}{V}}$ $\longrightarrow$ (1)

we know that, the charge density on the plates,

$$\rho = \frac{\text{Charge on the plates}}{\text{Area Covered by the plates}} = \frac{Q}{A}$$

$$Q = \rho A = 2 \times 10^{-6} \times 1 = 2 \times 10^{-6}\ C$$

$$= 2\mu C. \longrightarrow (2)$$

Also, we know that

$$E = V/d$$

$$\Rightarrow V = Ed.$$

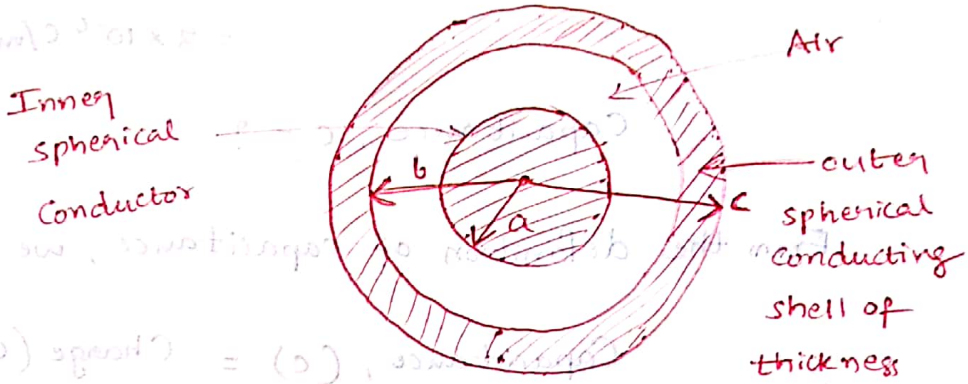
$$= 10^5 \times 1 \times 10^{-3} = 100\ V. \longrightarrow (3)$$

substituting the values of Q and V from equations (2) & (3) respectively, in equation (1), we get

$$\Rightarrow C = \frac{2 \times 10^{-6}}{100} = 0.02 \times 10^{-6} F$$

$$= 0.02\ \mu F.$$

(3) A concentric spherical conductor arrangement is shown in figure. If the capacitance of the arrangement is 0.1 nF and a is 10 cm. Find b.



Sol:- Given data

Capacitance,

$$C = 0.1\ nF$$

$$C = 0.1 \times 10^{-9}\ F$$

$$a = 10\,cm = 10 \times 10^{-2}\ m$$

$$= 0.1\ m$$

To calculate, $b = ?$

Figure shows that, two concentric spherical conductors of radii 'a' and 'c' placed concentrically and space between them is filled with dielectric material of relative permitivity ε_r. A Gaussian surface in the form of sphere of radius 'b' is concentric with these spheres.

The capacitance of spherical conductors is given by,

$$C = 4\pi\varepsilon\left[\frac{ac}{c-a}\right] F$$

$$\varepsilon = \varepsilon_0 \varepsilon_r \qquad [\text{As } \varepsilon_r = 1 \text{ for medium air}$$

$$\varepsilon_0 = 8.854 \times 10^{-12}\ F/m.]$$

$$\therefore \quad \varepsilon = 8.854 \times 10^{-12}$$

$$C = 4\pi \times 8.854 \times 10^{-12} \left[\frac{0.1\ c}{c-0.1} \right]$$

$$0.1 \times 10^{-9} = 4\pi \times 8.854 \times 10^{-12} \left[\frac{0.1c}{c-0.1} \right]$$

$$c - 0.1 = \frac{1.1120 \times 10^{-10} \times 0.1c}{0.1 \times 10^{-9}}$$

$\cancel{c = 11.2}$ $\cdot$ $C = 0.1125\ m$

$$\boxed{c = 11.25\ cm}$$

Given that, the thickness b/w outer spherical conductor and Gaussian spherical conductor is $(c-b)$ m. As the conductors are concentrically placed, so the thickness b/w sphere b and c and the thickness between a and b are same.

$\therefore$ $\cancel{b=a}$ $b - a = c - b$

$\Rightarrow 2b = a + c$

$$\Rightarrow b = \frac{a+c}{2}$$

$$\Rightarrow b = \frac{10 + 11.2}{2}$$

$$\Rightarrow b = \frac{21.2}{2}$$

$$\Rightarrow \boxed{b = 10.6\ cm}$$

March-06 (8M)

(4) Calculate the Capacitance of parallel plate capacitor with following details.

plate area = 100 sq. cm.

Dielectric $\varepsilon_{r_1} = 4$; $d_1 = 2$ mm

Dielectric $\varepsilon_{r_2} = 3$, $d_2 = 3$ mm

If 200 V is applied across the plates, what will be the voltage gradient across each dielectric ?

Sol:- Given data,

plate Area, $a = 100\,cm^2 = 100 \times 10^{-4}\,m^2 = 0.01\,m^2$

Relative permitivity dielectric-1, $\varepsilon_{r_1} = 4$

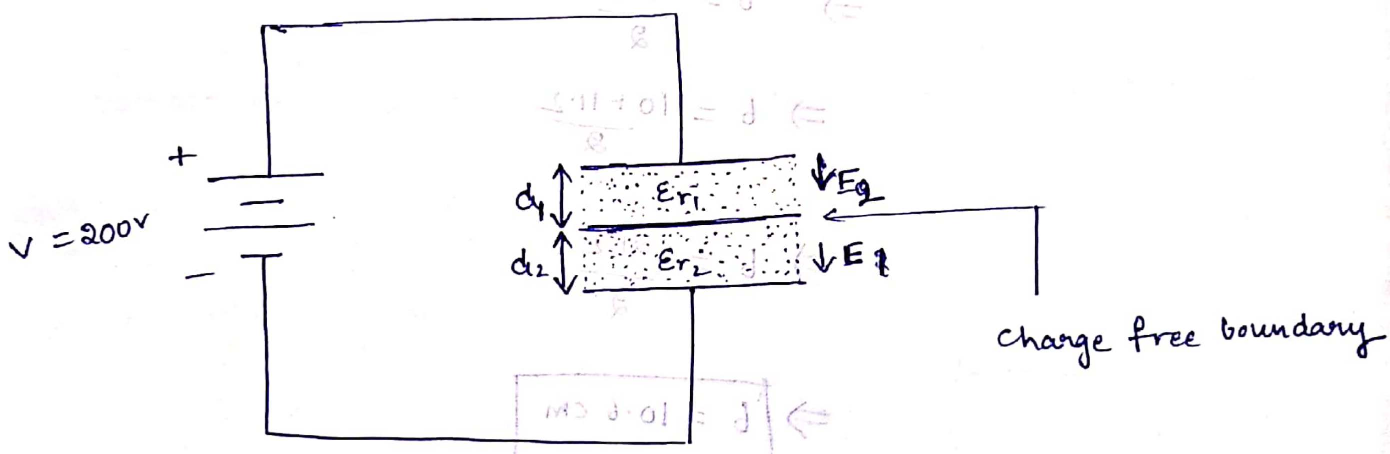
Relative permitivity dielectric-2, $\varepsilon_{r_2} = 3$

Thickness of dielectric-1, $d_1 = 2mm = 2 \times 10^{-3}\,m$.

Thickness of dielectric-2, $d_2 = 3mm = 3 \times 10^{-3}\,m$.

voltage applied across the plates, $V = 200$m.

Voltage gradient across each dielectric, i.e., E_1 and E_2 = ?



From the dielectric boundary conditions,

we have $D_{n_1} = D_{n_2}$

$$\Rightarrow \varepsilon_1 E_1 = \varepsilon_2 E_2 \quad [\because D = \varepsilon E]$$

$$\Rightarrow \varepsilon_0 \varepsilon_{r_1} E_1 = \varepsilon_0 \varepsilon_{r_2} E_2$$

$$\Rightarrow \boxed{E_1 = \frac{\varepsilon_{r_2} E_2}{\varepsilon_{r_1}} = \frac{3E_2}{4}} \longrightarrow (1)$$

If V_1 and V_2 are the potentials appeared across the dielectrics-1 and 2 respectively, then we have

$$V = V_1 + V_2$$

$$\Rightarrow 200 = E_1 d_1 + E_2 d_2$$

$$\Rightarrow 200 = \frac{3E_2}{4} \times (2\times10^{-3}) + E_2 \times (3\times10^{-3})$$

$$\Rightarrow 200 = 4.5\times10^{-3} E_2$$

$$\Rightarrow E_2 = \frac{200}{4.5\times10^{-3}} = 44.444\times10^{3} \text{ V/m}.$$

$$= 44.444 \text{ kV/m}.$$

Substituting the value of E_2 in eq (1), we get,

$$\Rightarrow E_1 = \frac{3}{4}(44.444) = 33.333 \text{ kV/m}.$$

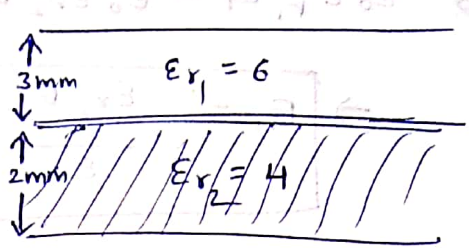
$\therefore$ voltage gradient across the dielectric-1 is $E_1 = 33.333$ kV/m

and across dielectric-2 is $E_2 = 44.444$ kV/m.

(5) A parallel plate capacitor has a plate area of 1.5 sq.m and a plate seperation of 5 mm. There are two dielectrics in between the plates. The first dielectric has a thickness of 3mm with a relative permitivity of 6 and the second has a thickness of 2 mm with relative permitivity 4. Find the capacitance.

Sol:-

$$C = \frac{Q}{V}$$



$$V = Ed$$

$$V = V_1 + V_2$$

$$= E_1 d_1 + E_2 d_2$$

$$= \frac{D}{\varepsilon_0 \varepsilon_{r_1}} d_1 + \frac{D}{\varepsilon_0 \varepsilon_{r_2}} \cdot d_2$$

$$V = \frac{Q}{A\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}}\right]$$

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}}\right]}$$

$$\boxed{C = \frac{A\varepsilon_0}{\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}}}}$$

Given data

plate Area, $a = 1.5\ m^2$

plate seperation, $d = 5mm = 5 \times 10^{-3}$ m.

First dielectric,

Relative permitivity, $\varepsilon_{r_1} = 6$.

Thickness, $d_1 = 3mm = 3 \times 10^{-3}$ m.

Second dielectric,

Relative permitivity, $\varepsilon_{r_2} = 4$

Thickness, $d_2 = 2mm = 2 \times 10^{-3}$ m.

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$$\text{Capacitance}, \; C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}}}$$

$$= \frac{8.854 \times 10^{-12} \times 1.5}{\frac{3 \times 10^{-3}}{6} + \frac{2 \times 10^{-3}}{4}}$$

$$= \frac{1.5 \times 8.854 \times 10^{-12}}{(0.5 + 0.5)10^{-3}}$$

$$= 13.281 \times 10^{-9} \, F$$

$$\boxed{C = 13.281 \; nF}$$

05 - 8M.

(6) The Construction of a paper capacitor is as follows. Alluminium foil of 100 cm^2 area is placed on both sides of paper of thickness 0.03 mm. If the dielectric ~~Construction~~ constant of paper is given as 3 and its dielectric breakdown strength is 200 kV/cm. what is the rating of the Capacitor?

Sol:- Given data,

Area of alluminium foil,

$$a = 100 \, cm^2$$
$$= 100 \times 10^{-4} \, m^2$$

paper thickness, $d = 0.03$ mm
$$= 0.03 \times 10^{-3} \, m.$$

Dielectric Constant (or) the relative permitivity of paper, $(\varepsilon_r) = 3$.

Dielektric breakdown strength, E_{max} = 200 kv / cm.

Rating of Capacitor = ?

We know that,

Capacitance, $C = \frac{\varepsilon a}{d} = \frac{\varepsilon_0 \varepsilon_r a}{d}$

$= \frac{8.854 \times 10^{-12} \times 3 \times 100 \times (10)^{-4}}{0.03 \times 10^{-3}}$

$= 8.854 \times 10^{-9} F$

$= 8.854 \, nF$.

Also, we know that,

Maximum Operating voltage,

$V_{max} = E_{max} \cdot d$

$= 200 \times 0.03$

$= 6 \, kv$.

∴ Rating of given paper Capacitor is 8.854 nF, 6 kv.

(7) A parallel plate capacitor consists of two square metal plates with 500 mm side and seperated by 10 mm. A slab of sulphur ($\varepsilon_r = 4$) 6 mm thick is placed on the lower plate and air gap of 4 mm. Find capacitance of Capacitor ?

Sol:- Given data,

parallel plate Capacitor with two square metal plates of each side, s = 500 mm $= 500 \times 10^{-3}$ m

Area of the square plates, $a = s^2 = (500 \times 10^{-3})^2 \, m^2$

Distance b/w the plates, d = 10 mm
$= 10 \times 10^{-3}$ m.

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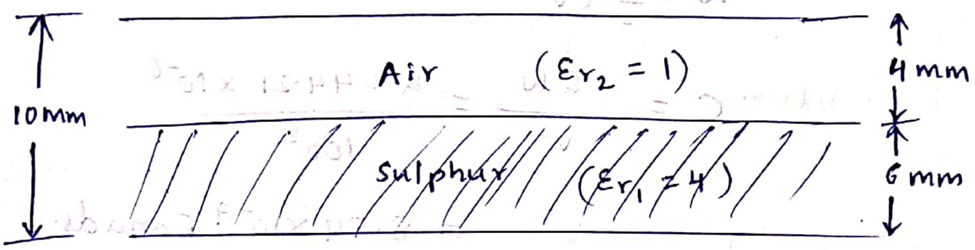
Thickness of sulphur slab, $t_1 = 6mm = 6 \times 10^{-3}$ m

Thickness of air gap, $t_2 = 4mm = 4 \times 10^{-3}$ m

permitivity of sulphur, $\varepsilon_{r_1} = 4$.

permitivity of air, $\varepsilon_{r_2} = 1$



$$C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}}}$$

$$= \frac{8.854 \times 500 \times 10^{-3} \times 500 \times 10^{-3}}{\frac{6 \times 10^{-3}}{4} + \frac{4 \times 10^{-3}}{1}}$$

$$\boxed{C = 0.4 \; nF}$$

(8) A parallel plate capacitor with a large plate area is situated in air. With a potential difference of 100V b/w the plates, the stored energy 44.21 μ Joule/unit area. Find the distance of seperation between the plates?

Sol:- Given data,

potential difference, $V = 100V$

stored energy, $W = 44.21$ μJ

$= 44.21 \times 10^{-6}$ J

Distance of seperation, $d = ?$

Given medium is air

$$\therefore \varepsilon = \varepsilon_0$$

we have,

Energy stored in a capacitor

$$W = \frac{1}{2} C V^2$$

$$C = \frac{2W}{V^2} = \frac{2 \times 44.21 \times 10^{-6}}{100^2}$$

$$= 8.84 \times 10^{-9} \text{ Farads.}$$

$\therefore$ Capacitance, $C = 8.84 \times 10^{-9}$ F

Assuming the area of the plates as unit area

$$A = 1 m^2$$

we have,

Capacitance, $C = \frac{\varepsilon A}{d}$

$$\Rightarrow C = \frac{A \varepsilon_0}{d} \quad (\because \varepsilon = \varepsilon_0)$$

Distance of seperation b/w the plates,

$$d = \frac{A \varepsilon_0}{C}$$

$$= \frac{1 \times 8.854 \times 10^{-12}}{8.84 \times 10^{-9}}$$

$$= 1 \times 10^{-3} m$$

$$d = 1mm$$

Distance of seperation b/w the plates

$$\boxed{d = 1mm}$$

(a) Obtain the fundamentals an expression for the Capacitance per unit area of a parallel-plate capacitor. If the plates are seperated by 1mm in air and have a potential difference of 1000 V. What is the energy stored per unit area?

Sol:

Given that

Distance, $d = 1\,mm = 1 \times 10^{-3}\,m$

potential difference, $V = 1000\,V$

Energy stored/unit area, $W_E = ?$

Capacitance, $C = \frac{\epsilon_0 A}{d}$ (As the medium is air, so $\epsilon = \epsilon_0$)

Capacitance/unit area, $\frac{C}{A} = \frac{\epsilon_0}{d}$

where $A \rightarrow$ Surface area of plate

$$\frac{C}{A} = \frac{8.854 \times 10^{-12}}{1 \times 10^{-3}}$$

$$\frac{C}{A} = 8.854 \times 10^{-9}\ F$$

Energy stored per unit area,

$$W_E = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 8.854 \times 10^{-9} \times (1000)^2$$

$$= 4.427 \times 10^{-3}\ J$$

(10) What is the Capacitance of a capacitor Consisting of two parallel plates of 30cm by 30cm, Seperated by 5mm in air? What is the energy stored by the Capacitor if it is charged to a potential

Sol Given data,

Dimensions of two plates = 30cm x 30cm

Distance between two plates, $d = 5\,mm$

Capacitance, $C = ?$

P.D b/w plates, $V = 500\,V$

Energy stored, $W_E = ?$

The Area of plates, $A = 30\,cm \times 30\,cm$

$$= 30 \times 10^{-2} \times 30 \times 10^{-2}$$

$$= 90 \times 10^{-3}\,m^2$$

The capacitance of the parallel plate Capacitor is

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 1 \times 90 \times 10^{-3}}{5 \times 10^{-3}}$$

$$= 159.375\,pF$$

We know that,

The energy stored by a Capacitor is

$$W_e = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \times 159.375 \times 10^{-12} \times (500)^2$$

$$= 19.922\,\mu J.$$